M^r:ABDELMOUTALEB

Fonctions Trigonométriques et hyperboliques

α	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
$\sin\alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1
$\cos\alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0
$\tg\alpha$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0	∞

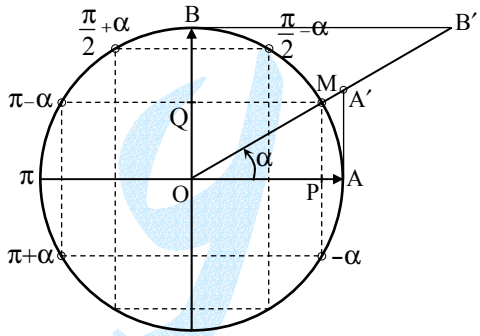
$\cos\alpha = \overline{OP}$ $\quad -1 \le \cos\alpha \le 1$

$\sin\alpha = \overline{OQ}$ $\quad -1 \le \sin\alpha \le 1$

$\tg\alpha = \overline{AA'} = \dfrac{\sin\alpha}{\cos\alpha} = \dfrac{1}{\cotg\alpha}$

$\cotg\alpha = \overline{BB'} = \dfrac{\cos\alpha}{\sin\alpha} = \dfrac{1}{\tg\alpha}$

$\sec\alpha = \dfrac{1}{\cos\alpha}$; $\operatorname{cosec}\alpha = \dfrac{1}{\sin\alpha}$



$\sin^2\alpha + \cos^2\alpha = 1$

$1 + \tg^2\alpha = \dfrac{1}{\cos^2\alpha}$

$1 + \cotg^2\alpha = \dfrac{1}{\sin^2\alpha}$

$\cos(\alpha + 2k\pi) = \cos\alpha$
$\sin(\alpha + 2k\pi) = \sin\alpha$
$\tg(\alpha + k\pi) = \tg\alpha$
$\cos(-\alpha) = \cos\alpha$
$\sin(-\alpha) = -\sin\alpha$
$\tg(-\alpha) = -\tg\alpha$
• k nombre relatif •

$\cos(\pi + \alpha) = -\cos\alpha$
$\sin(\pi + \alpha) = -\sin\alpha$
$\tg(\pi + \alpha) = \tg\alpha$
$\cos(\pi - \alpha) = -\cos\alpha$
$\sin(\pi - \alpha) = \sin\alpha$
$\tg(\pi - \alpha) = -\tg\alpha$

$\cos(\frac{\pi}{2} - \alpha) = \sin\alpha$
$\sin(\frac{\pi}{2} - \alpha) = \cos\alpha$
$\tg(\frac{\pi}{2} - \alpha) = \dfrac{1}{\tg\alpha}$
$\cos(\frac{\pi}{2} + \alpha) = -\sin\alpha$
$\sin(\frac{\pi}{2} + \alpha) = \cos\alpha$
$\tg(\frac{\pi}{2} + \alpha) = -\dfrac{1}{\tg\alpha}$

$\sin\alpha = 0 \Leftrightarrow \alpha = k\pi$
$\cos\alpha = 0 \Leftrightarrow \alpha = \frac{\pi}{2} + k\pi$
$\sin\alpha = 1 \Leftrightarrow \alpha = \frac{\pi}{2} + 2k\pi$
$\cos\alpha = 1 \Leftrightarrow \alpha = 2k\pi$
$\sin\alpha = -1 \Leftrightarrow \alpha = \frac{3\pi}{2} + 2k\pi$
$\cos\alpha = -1 \Leftrightarrow \alpha = \pi + 2k\pi$
• k nombre relatif •

$\cos2\alpha = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1$
$= 1 - 2\sin^2\alpha = \dfrac{1 - \tg^2\alpha}{1 + \tg^2\alpha}$

$\sin2\alpha = 2\sin\alpha\cos\alpha = \dfrac{2\tg\alpha}{1 + \tg^2\alpha}$

$2\cos^2\alpha = 1 + \cos2\alpha$
$2\sin^2\alpha = 1 - \cos2\alpha$
$4\cos^3\alpha = \cos3\alpha + 3\cos\alpha$
$4\sin^3\alpha = -\sin3\alpha + 3\sin\alpha$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\tg(\alpha+\beta) = \dfrac{\tg\alpha + \tg\beta}{1 - \tg\alpha\,\tg\beta}$

$\tg(\alpha-\beta) = \dfrac{\tg\alpha - \tg\beta}{1 + \tg\alpha\,\tg\beta}$

$\tg 2\alpha = \dfrac{2\tg\alpha}{1 - \tg^2\alpha}$

$\cos\alpha = \cos\beta \Leftrightarrow \begin{cases} \alpha = \beta + 2k\pi \\ V \\ \alpha = -\beta + 2k\pi \end{cases}$

$\sin\alpha = \sin\beta \Leftrightarrow \begin{cases} \alpha = \beta + 2k\pi \\ V \\ \alpha = \pi - \beta + 2k\pi \end{cases}$

$\tg\alpha = \tg\beta \Leftrightarrow \alpha = \beta + k\pi$
• k nombre relatif •

$\cos\alpha = \dfrac{e^{i\alpha} + e^{-i\alpha}}{2}$

$\sin\alpha = \dfrac{e^{i\alpha} - e^{-i\alpha}}{2i}$

$e^{i\alpha} = \cos\alpha + i\sin\alpha$
$e^{-i\alpha} = \cos\alpha - i\sin\alpha$
$e = 2{,}718281828\ldots$
i nbre imag: $i^2 = -1$

$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$
$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$
$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$

$\tg\alpha\,\tg\beta = \dfrac{\tg\alpha + \tg\beta}{\cotg\alpha + \cotg\beta} = \dfrac{\tg\beta - \tg\alpha}{\cotg\alpha - \cotg\beta}$

$a\cos\alpha + b\sin\alpha = \sqrt{a^2+b^2}\,\sin(\alpha+\theta)$

$\sin\theta = \dfrac{a}{\sqrt{a^2+b^2}}$; $\cos\theta = \dfrac{b}{\sqrt{a^2+b^2}}$

$a\cos\alpha + b\sin\alpha = \sqrt{a^2+b^2}\,\cos(\alpha-\theta)$

$\cos\theta = \dfrac{a}{\sqrt{a^2+b^2}}$; $\sin\theta = \dfrac{b}{\sqrt{a^2+b^2}}$

$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$
$\sin\alpha - \sin\beta = 2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$
$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$
$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

$\tg\alpha + \tg\beta = \dfrac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta}$

$\tg\alpha - \tg\beta = \dfrac{\sin(\alpha-\beta)}{\cos\alpha\cos\beta}$

$\operatorname{ch}\alpha = \dfrac{e^{\alpha} + e^{-\alpha}}{2}$; $\operatorname{sh}\alpha = \dfrac{e^{\alpha} - e^{-\alpha}}{2}$

$\operatorname{th}\alpha = \dfrac{\operatorname{sh}\alpha}{\operatorname{ch}\alpha} = \dfrac{e^{\alpha} - e^{-\alpha}}{e^{\alpha} + e^{-\alpha}}$

$\operatorname{cth}\alpha = \dfrac{\operatorname{ch}\alpha}{\operatorname{sh}\alpha} = \dfrac{e^{\alpha} + e^{-\alpha}}{e^{\alpha} - e^{-\alpha}}$

$\operatorname{ch}\alpha + \operatorname{sh}\alpha = e^{\alpha}$; $\operatorname{ch}\alpha - \operatorname{sh}\alpha = e^{-\alpha}$
$\operatorname{ch}^2\alpha - \operatorname{sh}^2\alpha = 1$; $\operatorname{th}\alpha\,\operatorname{cth}\alpha = 1$
$\operatorname{sh}(-\alpha) = -\operatorname{sh}\alpha$; $\operatorname{ch}(-\alpha) = \operatorname{ch}\alpha$
$\operatorname{th}(-\alpha) = -\operatorname{th}\alpha$; $\operatorname{cth}(-\alpha) = -\operatorname{cth}\alpha$

$\operatorname{sh}(\alpha \pm \beta) = \operatorname{sh}\alpha\operatorname{ch}\beta \pm \operatorname{ch}\alpha\operatorname{sh}\beta$
$\operatorname{ch}(\alpha \pm \beta) = \operatorname{ch}\alpha\operatorname{ch}\beta \pm \operatorname{sh}\alpha\operatorname{sh}\beta$

$\operatorname{th}(\alpha \pm \beta) = \dfrac{\operatorname{th}\alpha \pm \operatorname{th}\beta}{1 \pm \operatorname{th}\alpha\operatorname{th}\beta}$

$\operatorname{sh}2\alpha = 2\operatorname{sh}\alpha\operatorname{ch}\alpha = \dfrac{2\operatorname{th}\alpha}{1 - \operatorname{th}^2\alpha}$

$\operatorname{ch}2\alpha = \operatorname{ch}^2\alpha + \operatorname{sh}^2\alpha = 2\operatorname{sh}^2\alpha + 1$
$= 2\operatorname{ch}^2\alpha - 1 = \dfrac{1 + \operatorname{th}^2\alpha}{1 - \operatorname{th}^2\alpha}$

$\operatorname{th}2\alpha = \dfrac{2\operatorname{th}\alpha}{1 + \operatorname{th}^2\alpha}$; $\operatorname{cth}2\alpha = \dfrac{1 + \operatorname{cth}^2\alpha}{2\operatorname{cth}\alpha}$

$\operatorname{sh}\alpha \pm \operatorname{sh}\beta = 2\operatorname{sh}\left(\frac{\alpha \pm \beta}{2}\right)\operatorname{ch}\left(\frac{\alpha \mp \beta}{2}\right)$
$\operatorname{ch}\alpha + \operatorname{ch}\beta = 2\operatorname{ch}\left(\frac{\alpha+\beta}{2}\right)\operatorname{ch}\left(\frac{\alpha-\beta}{2}\right)$
$\operatorname{ch}\alpha - \operatorname{ch}\beta = 2\operatorname{sh}\left(\frac{\alpha+\beta}{2}\right)\operatorname{sh}\left(\frac{\alpha-\beta}{2}\right)$

$\operatorname{th}\alpha \pm \operatorname{th}\beta = \dfrac{\operatorname{sh}(\alpha \pm \beta)}{\operatorname{ch}\alpha\operatorname{ch}\beta}$

$(\operatorname{ch}\alpha \pm \operatorname{sh}\alpha)^n = \operatorname{ch}n\alpha \pm \operatorname{sh}n\alpha$

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