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BAC2020

larbibelabidi @ gmail.com

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1 2019 16

1

$u_n > 1$ n

(u_n) $u_0 = 13$ $u_{n+1} = \frac{1}{5}u_n + \frac{4}{5}$

$n = 0$ u_0 1 $u_0 = 13$

n ∈ u_n 1 n ∈ u_{n+1} 1

u_{n+1} 1 $\frac{1}{5}u_n + \frac{4}{5}$ 1 $\frac{1}{5}u_n$ $\frac{1}{5}$ u_n 1

(u_n)

(u_n) $u_{n+1} - u_n$

$u_{n+1} - u_n = \frac{1}{5}u_n + \frac{4}{5} - u_n = -\frac{4}{5}(u_n - 1)$

u_n 1 (u_n)

(u_n)

2

(v_n)

(v_n) q $v_{n+1} - v_n = q$ n ∈

$v_n = \ln(u_n - 1)$ $v_{n+1} = \ln(u_{n+1} - 1)$

$$v_{n+1} - v_n = \ln(u_{n+1} - 1) - \ln(u_n - 1) = \ln\left(\frac{u_{n+1} - 1}{u_n - 1}\right) = \ln\left(\frac{\frac{1}{5}(u_n - 1)}{(u_n - 1)}\right) = \ln\frac{1}{5}$$

(v_n) $\ln\frac{1}{5}$ $v_0 = \ln(u_0 - 1) = \ln(12)$

3

n v_n n $u_n = 1 + \frac{12}{5^n}$ $\lim_{x \to +\infty} u_n$

$v_n = v_0 + n.r$ n

$$v_n = \ln(12) + n\ln\frac{1}{5} = \ln\left(\frac{1}{5}\right)^n + \ln(12) = \ln\left(\frac{12}{5^n}\right)$$

$v_n = \ln(u_n - 1)$

$$u_n = e^{v_n} + 1 = e^{\ln(\frac{12}{5^n})} + 1 = 1 + \frac{12}{5^n} \qquad (u_n - 1) = e^{v_n}$$

$$\lim_{x\to+\infty} 5^n = +\infty \qquad \lim_{x\to+\infty} \frac{12}{5^n} = 0 \qquad \lim_{x\to+\infty} u_n = 1 + \lim_{x\to+\infty} \frac{12}{5^n} = 1$$

$(u_0 - 1)(u_1 - 1) \times \ldots\ldots \times (u_n - 1) = \left(\frac{12}{5^{\frac{n}{2}}}\right)^{n+1}$ **n** **4**

$$P_n = (u_0 - 1)(u_1 - 1) \times \ldots\ldots \times (u_n - 1)$$

$$u_n - 1 = \frac{12}{5^n}$$

$$P_n = (u_0 - 1)(u_1 - 1) \times \ldots\ldots \times (u_n - 1) = \frac{12}{5^0} \cdot \frac{12}{5^1} \ldots\ldots \frac{12}{5^n} = \frac{12.12\ldots\ldots12}{5^{0+1+2+\ldots\ldots+n}} = \frac{12^{n+1}}{5^{\frac{(n+1)n}{2}}} = \left(\frac{12}{5^{\frac{n}{2}}}\right)^{n+1}$$

2 2019 17

$[4;7[$ **f** 1

$f(x) = \sqrt{x+2} + 4$ $[4;7[$ f

$[4;7[$ f $f'(x)$ 0 $f'(x) = \frac{1}{2\sqrt{x+2}}$

$f(x) \in [4;7[$ $x \in [4;7[$

$[4;7[$ f $f(x) \in [f(4);f(7)[$ $x \in [4;7[$

$f(7) = 7$ $f(4) = \sqrt{6} + 4$ $f(x) \in \left[-4;-\frac{13}{12}\right]$

$[\sqrt{6}+4;7[$ $[4;7[$ $f(x) \in [\sqrt{6}+4;7[$

$f(x) - x = \frac{-x^2 + 9x - 14}{x - 4 + \sqrt{x+2}}$ $[4;7[$ **x** **2**

$[4;7[$ **x**

$$f(x) - x = \sqrt{x+2} - x + 4 = \frac{\left(\sqrt{x+2} - (x-4)\right)\left(\sqrt{x+2} + (x-4)\right)}{\left(\sqrt{x+2} + (x-4)\right)}$$

$$= \frac{x + 2 - (x-4)^2}{\left(\sqrt{x+2} + (x-4)\right)} = \frac{-x^2 + 9x - 14}{x - 4 + \sqrt{x+2}}$$

$f(x) - x > 0$ $x \in [4;7[$

$f(x) - x = \frac{-x^2 + 9x - 14}{x - 4 + \sqrt{x+2}}$ $x \in [4;7[$

$-x^2+9x-14$ $f(x)-x$

$x=7$ $x=2$ $-x^2+9x-14=0$

$x\in[4;7[$ $f(x)-x>0$ $x\in[2;7[$ $-x^2+9x-14>0$

$4\le u_n<7$ **n** 3

P(0)

$u_0=4$ $4\le u_0<7$ n=0

$4\le u_{n+1}<7$ P(n+1) $4\le u_n<7$ P(n)

f $f(4)\le f(u_n)<f(7)$ $4\le u_n<7$

1 $4\le u_{n+1}<7$

n $4\le u_n<7$

(u_n)

2 $f(u_n)-u_n=u_{n+1}-u_n=\dfrac{-u_n{}^2+9u_n-14}{u_n-4+\sqrt{u_n+2}}>0$

$u_{n+1}-u_n>0$ (u_n)

7 (u_n)

$7-u_{n+1}\le\dfrac{1}{4}(7-u_n)$ **n** 4

$$7-u_{n+1}=3-\sqrt{u_n+2}=\frac{(3-\sqrt{u_n+2})(3+\sqrt{u_n+2})}{3+\sqrt{u_n+2}}=\frac{(3)^2-u_n-2}{3+\sqrt{u_n+2}}=\frac{(7-u_n)}{3+\sqrt{u_n+2}}$$

$\dfrac{(7-u_n)}{3+\sqrt{u_n+2}}\le\dfrac{(7-u_n)}{4}$ $7-u_{n+1}\le\dfrac{1}{4}(7-u_n)$

$u_n<7$ $\dfrac{(7-u_n)}{3+\sqrt{u_n+2}}\le\dfrac{(7-u_n)}{4}$ $\dfrac{1}{3+\sqrt{u_n+2}}\le\dfrac{1}{4}$ $3+\sqrt{u_n+2}\ge4$

$7-u_{n+1}\le\dfrac{1}{4}(7-u_n)$ n

$\lim\limits_{x\to+\infty}u_n$ $0<7-u_n\le3\left(\dfrac{1}{4}\right)^n$ **n**

$7-u_{n+1}\le\dfrac{1}{4}(7-u_n)$

$7-u_2\le\dfrac{1}{4}(7-u_1)$ n= 1 $7-u_1\le\dfrac{1}{4}(7-u_0)$ n=0

$7-u_n\le\dfrac{1}{4}(7-u_{n-1})$ n 1

$$7-u_n \le \left(\frac{1}{4}\right)^n (4-u_0)$$

1 $7-u_n \le 3\left(\frac{1}{4}\right)^n$ n

2 $7-u_n > 0$ $u_n < 7$

n $0 < 7-u_n \le 3\left(\frac{1}{4}\right)^n$ 2 1

$\lim_{x\to+\infty} u_n$

$\lim_{x\to+\infty} u_n = 7$ $\lim_{x\to+\infty}(7-u_n)=0$ $\lim_{x\to+\infty} 3\left(\frac{1}{4}\right)^n = 0$ $0 < 7-u_n \le 3\left(\frac{1}{4}\right)^n$

1 2018 18

$u_n > -2$ n 1

$u_{n+1} = 1 - \frac{9}{u_n+5}$:n $u_0 = 1$ (u_n)

P(0)

$u_0 = 1$ $u_0 > -2$ n=0

$u_{n+1} > -2$ P(n+1) $u_n > -2$ P(n)

$\frac{9}{u_n+5}$ 3 $\frac{u_n+5}{9} > \frac{1}{3}$ $u_n+5>3$ $u_n > -2$

$u_{n+1} > -2$ $1-\frac{9}{u_n+5}$ -2 $-\frac{9}{u_n+5}$ -3

n $u_n > -2$

(u_n)

n $u_{n+1}-u_n$ (u_n)

$$u_{n+1}-u_n = 1-\frac{9}{u_n+5}-u_n = \frac{-u^2{}_n-4u_n-4}{u_n+5} = \frac{-(u_n+2)^2}{u_n+4} \quad 0$$

(u_n)

2 (u_n)

(u_n) (u_n)

$\frac{1}{3}$ (v_n) 2

$$v_n = \frac{1}{u_n + 2} \quad n$$

$$n \in \qquad v_{n+1} - v_n = \frac{1}{3} \qquad \frac{1}{3} \qquad (v_n)$$

$$u_{n+1} = 1 - \frac{9}{u_n + 5} = \frac{u_n - 4}{u_n + 5} \qquad v_{n+1} = \frac{1}{u_{n+1} + 2} = \frac{1}{\frac{u_n - 4}{u_n + 5} + 2} = \frac{u_n + 5}{3(u_n + 2)} \qquad v_n = \frac{1}{u_n + 2}$$

$$v_{n+1} - v_n = \frac{u_n + 5}{3(u_n + 2)} - \frac{1}{u_n + 2} == \frac{u_n + 2}{3(u_n + 2)} = \frac{1}{3}$$

$$v_0 = \frac{1}{u_0 + 2} = \frac{1}{3} \qquad \frac{1}{3} \qquad (v_n)$$

$\lim\limits_{x \to +\infty} u_n$ u_n v_n **n** **3**

$$v_n = \frac{1}{3}(n+1) \qquad v_n = v_0 + nr \qquad n$$

$$u_n = \frac{1}{v_n} - 2 = \frac{3}{n+1} - 2 = \frac{-2n+1}{n+1} \qquad u_n + 2 = \frac{1}{v_n} \qquad v_n = \frac{1}{u_n + 2}$$

$$\lim_{x \to +\infty} u_n = \lim_{x \to +\infty} \frac{-2n+1}{n+1} = -2$$

$u_0.v_0 + u_1.v_1 + ... + u_n.v_n = \frac{1}{3}(1 - n^2)$ **n** **4**

$$v_n.u_n = 1 - 2v_n \qquad v_n.u_n + 2.v_n = 1 \qquad v_n(u_n + 2) = 1 \qquad v_n = \frac{1}{u_n + 2}$$

$$S_n = u_0.v_0 + u_1.v_1 + ... + u_n.v_n$$

$$S_n = (1 - 2v_0) + (1 - 2v_1) + ... + (1 - 2v_n) = (n+1) - 2(v_0 + v_1 + ... + v_n)$$

$$v_0 + v_1 + ... + v_n = \frac{n+1}{2}(v_0 + v_n) = \frac{n+1}{2}(\frac{n+2}{3})$$

$$S_n = (n+1) - 2(\frac{n+1}{2})(\frac{n+2}{3}) = \frac{3(n+1) - (n+1)(n+2)}{3} = \frac{1}{3}(1+n)(1-n) = \frac{1}{3}(1 - n^2)$$

1 2018 19

u_3 u_2 u_1 **1**

$$u_{n+1} = u_n + \ln\left(\frac{2n+3}{2n+1}\right) \qquad n \qquad u_0 = 0 \qquad (u_n)$$

$$u_3 = u_2 + \ln\left(\frac{7}{5}\right) = \ln 7 \quad u_2 = u_1 + \ln\left(\frac{5}{3}\right) = \ln 5 \quad u_1 = u_0 + \ln(3) = \ln 3$$

$\frac{2n+3}{2n+1} > 1$ **n** **2**

$\frac{2n+3}{2n+1} > 1$ $\frac{2n+3}{2n+1} - 1 = \frac{2}{2n+1} > 0$

(u_n)

$\ln\left(\frac{2n+3}{2n+1}\right) > 0$ $\frac{2n+3}{2n+1} > 1$

$u_{n+1} - u_n > 0$ $u_{n+1} - u_n = \ln\left(\frac{2n+3}{2n+1}\right)$ $u_{n+1} = u_n + \ln\left(\frac{2n+3}{2n+1}\right)$

(u_n)

$e^{u_n} = v_n$ **n** **3**

P(0)

$v_0 = 2(0)+1 = 1$ $e^{u_0} = e^0 = v_0$ n=0

$e^{u_{n+1}} = v_{n+1}$ P(n+1) $e^{u_n} = v_n$ P(n)

$e^{u_{n+1}} = e^{u_n + \ln\left(\frac{2n+3}{2n+1}\right)} = e^{u_n}.e^{\ln\left(\frac{2n+3}{2n+1}\right)} = v_n.\frac{2n+3}{2n+1} = 2n+3 = v_{n+1}$ $u_{n+1} = u_n + \ln\left(\frac{2n+3}{2n+1}\right)$

n $e^{u_n} = v_n$

$\lim\limits_{x\to+\infty} u_n$ **n** (u_n)

$\lim\limits_{x\to+\infty} u_n = \lim\limits_{x\to+\infty} \ln v_n = \lim\limits_{x\to+\infty} (2n+1) = +\infty$ $u_n = \ln v_n = \ln(2n+1)$ $e^{u_n} = v_n$

T S_n **4**

$S_n = \ln\left(\frac{v_1}{v_0}\right) + \ln\left(\frac{v_2}{v_1}\right) + \ldots\ldots + \ln\left(\frac{v_n}{v_{n-1}}\right) = \ln\left(\frac{v_1}{v_0}\right).\left(\frac{v_2}{v_1}\right)\ldots\ldots\ldots\ln\left(\frac{v_n}{v_{n-1}}\right) = \ln\left(\frac{v_n}{v_0}\right) = \ln v_n = \ln(2n+1)$

$T = e^{u_{1439}} + e^{u_{1440}} + \ldots + e^{u_{2018}} = v_{1439} + v_{1440} + \ldots\ldots + v_{2018}$ $e^{u_n} = v_n$

1 2 (v_n) $v_n = 2n+1$

$v_{2018} = 2(2018)+1 = 4037$ $v_{1439} = 2(1439)+1 = 2879$ $T = \frac{2018-1439+1}{2}(v_{1439} + v_{2018}) = 2005640$

1 2017 20

$0 \le u_n \le 1$ **n** **1**

P(0)

$u_0 = \frac{1}{4}$ 0 u_0 1 n=0

0 u_{n+1} 1 P(n+1) 0 u_n 1 P(n)

$\frac{4}{10}$ $\frac{u_n+4}{10}$ $\frac{5}{10}$ 4 u_n+4 5 0 u_n 1

$-\frac{10}{4}$ $-\frac{10}{u_n+4}$ $-\frac{10}{5}$ $\frac{10}{5}$ $\frac{10}{u_n+4}$ $\frac{10}{4}$

0 u_n 1 $\frac{1}{2}$ u_{n+1} 1 $3-\frac{10}{4}$ $3-\frac{10}{u_n+4}$ $3-\frac{10}{5}$

n 0 u_n 1

(u_n)

n $u_{n+1}-u_n$ (u_n)

$$u_{n+1}-u_n=3-\frac{10}{u_n+4}-u_n=\frac{(3-u_n)(u_n+4)-10}{u_n+4}=\frac{(1-u_n)(u_n-2)}{u_n+4}$$

$-u^2_n-u_n+2$

0 u_n 1 $u_n=2$ $u_n=1$ $-u^2_n-u_n+2=0$

$-u^2_n-u_n+2$

u_n	$-\infty$	1	2	$+\infty$
		0	0	
		(u_n) (u_n)		(u_n)

(u_n) 0 u_n 1

1 (u_n)

$\frac{5}{2}$ (v_n) **2**

$n\in$ $v_{n+1}=\frac{5}{2}v_n$ $\frac{5}{2}$ (v_n)

$$v_{n+1}=\frac{u_{n+1}+2}{1-u_{n+1}}=\frac{\left(3-\frac{10}{u_n+4}\right)+2}{1-\left(3-\frac{10}{u_n+4}\right)}=\frac{5(u_n+2)}{2(1-u_n)}=\frac{5}{2}v_n \qquad v_n=\frac{u_n+2}{1-u_n}$$

n v_n

$v_n=3.\left(\frac{5}{2}\right)^n$ $v_n=v_0.q^n$ $v_0=\frac{u_0+2}{1-u_0}=3$ $q=\frac{5}{2}$

$\lim\limits_{x\to+\infty}u_n$ $u_n=1-\frac{3}{v_n+1}$ n

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$$v_n - u_n.v_n = u_n + 2 \qquad v_n(1-u_n) = u_n + 2 \qquad v_n = \frac{u_n + 2}{1-u_n}$$

$$u_n = \frac{v_n - 2}{1+v_n} = \frac{v_n + 1 - 3}{1+v_n} = 1 - \frac{3}{1+v_n} \qquad v_n - 2 = u_n(1+v_n)$$

$$\lim_{x\to+\infty} u_n = \lim_{x\to+\infty} 1 - \frac{3}{1+v_n} = 1 \qquad \left(\frac{5}{2}\right) \quad 1 \qquad \lim_{x\to+\infty} v_n = \lim_{x\to+\infty} 3.\left(\frac{5}{2}\right)^n = +\infty$$

2 2017 21

$[-4;1]$ **f** **I**

$$f(x) = \frac{3x-16}{x+11} \qquad [-4;1] \qquad f$$

$$f'(x) = \frac{3(x+11) - 1(3x-16)}{(x+11)^2} = \frac{49}{(x+11)^2}$$

$[-4;1]$ f $f'(x)$ 0

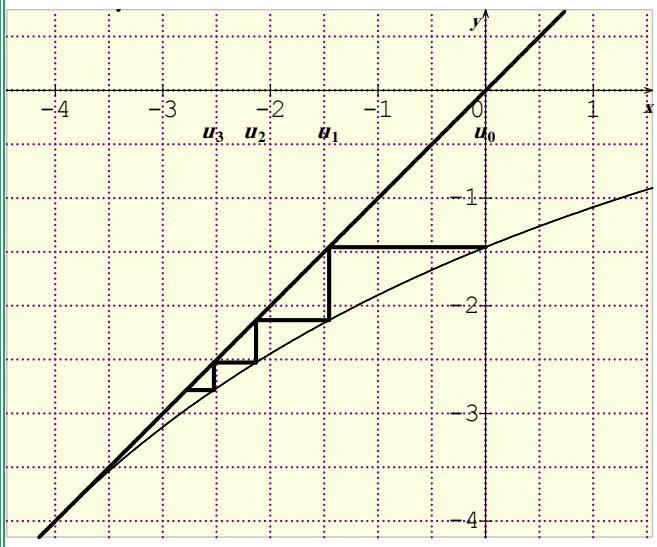
$f(x) \in [-4;1]$ $x \in [-4;1]$

$[-4;1]$ f $f(x) \in [f(-4);f(1)]$ $x \in [-4;1]$

$$f(1) = \frac{-13}{12} \qquad f(-4) = -4 \qquad f(x) \in \left[-4;-\frac{13}{12}\right]$$

$[-4;1]$ $\left[-4;-\frac{13}{12}\right]$ $f(x) \in [-4;1]$

u_3 u_2 u_1 u_0 1 **II**



(u_n) (u_n)

4

$-4 \le u_n \le 0$ $n \in$ 2

P(0)

$u_0 = 0$ $-4 \le u_0 \le 0$ n=0

$-4 \le u_n \le 0$ P(n)

$-4 \le u_{n+1} \le 0$ P(n+1)

$f(-4) \le f(u_n)$ $f(0)$ $-4 \le u_n \le 0$

$[-4;0]$ $\left[-4;-\frac{16}{11}\right]$ $-4 \le u_{n+1} \le -\frac{16}{11}$ f

$n \in$ $-4 \le u_n \le 0$

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(u_n)

$$u_{n+1}-u_n=f(u_n)-u_n=\frac{3u_n-16}{u_n+11}-u_n=\frac{-u_n^2-8u_n-16}{u_n+11}=\frac{-(u_n+4)^2}{u_n+11}$$

u_n	$-\infty$	4	$+\infty$
	0		
	(u_n)		(u_n)

(u_n) $-4\le u_n\le 0$

2 (v_n) $\frac{1}{7}$

$n\in$ $v_{n+1}-v_n=\frac{1}{7}$ $\frac{1}{7}$ (v_n)

$$v_n=\frac{1}{u_n+4} \qquad v_n(u_n+4)=1 \qquad v_n\times u_n=1-4v_n$$

$$v_{n+1}-v_n=\frac{1}{u_{n+1}+4}-\frac{1}{u_n+4}=\frac{u_n+11}{7(u_n+4)}-\frac{1}{u_n+4}=\frac{u_n+4}{7(u_n+4)}=\frac{1}{7}$$

S

$v_n\times u_n=1-4v_n$ $S=v_0.u_0+v_1.u_1+\ldots+v_{2016}.u_{2016}$

$$S=(1-4v_0)+(1-4v_1)+\ldots+(1-4v_{2016})=2017-4(v_0+v_1+\ldots.+v_{2016})$$

$$(v_0+v_1+\ldots.+v_{2016})=\frac{2017}{2}(v_0+v_{2016})$$

$v_{2016}=\frac{1}{4}+288$ $v_0=\frac{1}{4}$ $S=2017-2.2017(\frac{1}{2}+288)=-1161792$

1 **2017** **22**

1 v_1 u_1

$$\begin{cases} v_0=6 \\ v_{n+1}=\frac{3}{4}v_n+1 \end{cases} \qquad \begin{cases} u_0=1 \\ u_{n+1}=\frac{3}{4}u_n+1 \end{cases}$$ (v_n) (u_n)

$$v_1=\frac{3}{4}v_0+1=\frac{11}{2} \qquad u_1=\frac{3}{4}u_0+1=\frac{7}{4}$$

2 $u_{n+1}-u_n$ $u_{n+2}-u_{n+1}$

$$u_{n+2}-u_{n+1}=u_{n+1}=\frac{3}{4}u_{n+1}-\frac{3}{4}u_n=\frac{3}{4}(u_{n+1}-u_n)$$

(v_n) (u_n)

(u_n)

n $u_{n+1}-u_n$ 0 (u_n)

P(0)

$u_1-u_0=\frac{7}{4}-1=\frac{3}{4}$ u_1-u_0 0 n=0

$u_{n+1}-u_n$ 0 P(n)

$u_{n+2}-u_{n+1}$ 0 P(n+1)

$u_{n+2}-u_{n+1}=\frac{3}{4}(u_{n+1}-u_n)$ $u_{n+2}-u_{n+1}$ 0 $u_{n+1}-u_n$ 0

n $u_{n+1}-u_n$ 0

(v_n)

n $v_{n+1}-v_n$ 0 (v_n)

P(0)

$v_1-v_0=\frac{11}{2}-6=-\frac{1}{2}$ $v_{n+1}-v_n$ 0 n=0

$v_{n+2}-v_{n+1}$ 0 P(n+1) $v_{n+1}-v_n$ 0 P(n)

$v_{n+2}-v_{n+1}=\frac{3}{4}(v_{n+1}-v_n)$ $v_{n+2}-v_{n+1}$ 0 $v_{n+1}-v_n$ 0

n $v_{n+1}-v_n$ 0

w_0 q (w_n) 3

$w_{n+1}=q.w_n$ n (w_n)

$w_n=u_n-v_n$ (w_n)

$w_{n+1}=u_{n+1}-v_{n+1}=\frac{3}{4}(u_n-v_n)=\frac{3}{4}w_n$

$w_0=u_0-v_0=-5$ $q=\frac{3}{4}$ (w_n)

(v_n) (u_n) 4

$\lim_{x\to+\infty}(u_n-v_n)=0$ (v_n) (u_n) (v_n) (u_n)

2 (v_n) (v_n) (u_n)

$\frac{3}{4}$ 1 $\lim_{x\to+\infty}(u_n-v_n)=\lim_{x\to+\infty}w_n=-5\lim_{x\to+\infty}\left(\frac{3}{4}\right)^n=0$

I (u_n) α

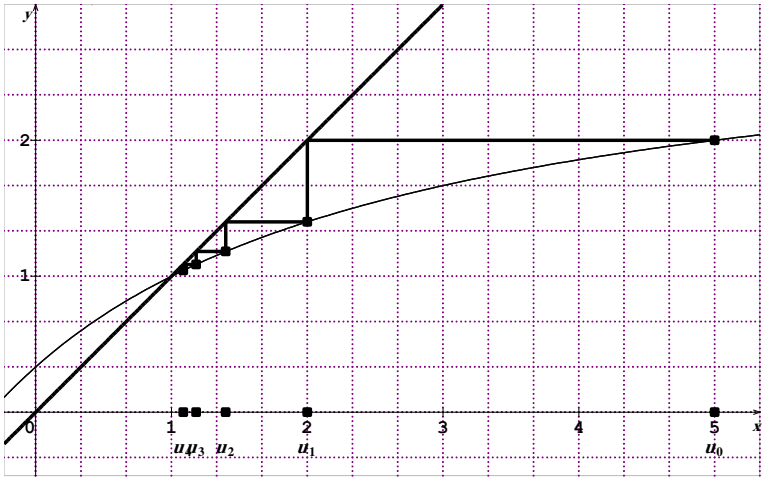
$u_0 = \alpha$ u_0 (u_n)

$u_{n+1} = f(u_n)$: n

n $u_{n+1} = u_n = u_0 = \alpha$ (u_n)

$u_{n+1} - u_n = \frac{3\alpha+1}{\alpha+3} - \alpha = \frac{-\alpha^2+1}{\alpha+3} = 0$ $\alpha = 1$ $\alpha = -1$ $-\alpha^2+1=0$

II 1 u_0 u_1 u_2 u_3



1 (u_n)

2 (v_n) $\frac{1}{2}$

(v_n) $\frac{1}{2}$ n $v_{n+1} = \frac{1}{2}.v_n$

$v_{n+1} = \frac{u_{n+1}-1}{u_{n+1}+1} = \frac{\frac{3u_n+1}{u_n+3}-1}{\frac{3u_n+1}{u_n+3}+1} = \frac{2(u_n-1)}{4(u_n+1)} = \frac{1}{2}v_n$ $v_n = \frac{u_n-1}{u_n+1}$

$v_0 = \frac{u_0-1}{u_0+1} = \frac{2}{3}$ (v_n)

$\lim_{x\to+\infty} u_n$ u_n v_n n

$v_n = \frac{2}{3}(\frac{1}{2})^n$ $v_n = v_0(q)^n$ n

$u_n + 1 = \frac{2}{1-v_n}$ $\frac{2}{u_n+1} = 1 - v_n$ $v_n = 1 - \frac{2}{u_n+1}$ $v_n = \frac{u_n-1}{u_n+1}$

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$\lim\limits_{x\to+\infty} v_n = 0$ $\lim\limits_{x\to+\infty} u_n = 1$ $u_n = \dfrac{2}{1-\frac{2}{3}(\frac{1}{2})^n} - 1$ $u_n = \dfrac{2}{1-v_n} - 1$

S_n n 3

$$S_n = v_n + v_{n+1} + ...v_{n+2016}$$

$$S_n = v_n\left[\frac{1-q^{2017}}{1-q}\right] = \frac{2}{3}\left(\frac{1}{2}\right)^n\left[\frac{1-\left(\frac{1}{2}\right)^{2017}}{1-\left(\frac{1}{2}\right)}\right] = \frac{4}{3}\left(\frac{1}{2}\right)^n\left(1-\left(\frac{1}{2}\right)^{2017}\right)$$

S_n' n

$$S_n' = \frac{1}{u_n+1} + \frac{1}{u_{n+1}+1} + ... + \frac{1}{u_{n+2016}+1}$$

$\dfrac{1}{u_n+1} = \dfrac{1}{2}(1-v_n)$ $\dfrac{2}{u_n+1} = 1-v_n$

$S_n' = \dfrac{1}{2}\left[(2017) - S_n\right]$ $S_n' = \dfrac{1}{2}\left[(1-v_n) + (1-v_{n+1}) + ... + (1-v_{n+2016})\right]$

1 2016 24

$\lim\limits_{x\to+\infty} f(x)$ 1 I

$\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} \sqrt{2x} = +\infty$ $f(x) = \sqrt{2x+8}$ $[0;+\infty[$ f

f

$f'(x)$

$f'(x) = \dfrac{(2x+8)'}{2\sqrt{2x+8}} = \dfrac{1}{\sqrt{2x+8}}$ $[0;+\infty[$ f

f $f'(x)$ 0

x	0	$+\infty$
$f'(x)$		
$f(x)$	$f(0)$ ↗	$+\infty$

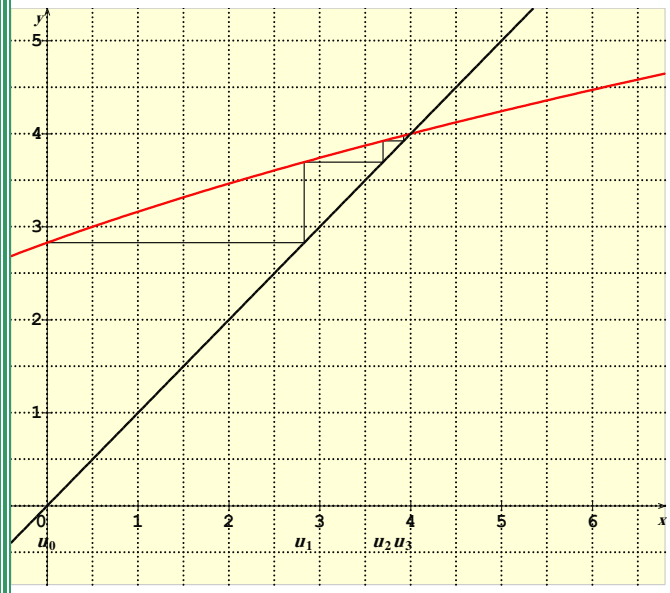
(Δ) (C) 2

$f(x) = x$ (Δ) (C)

$x^2 - 2x - 8 = 0$ $[f(x)]^2 = x^2$ $f(x) = x$

$x=-2$ $x=4$ $x^2-2x-8=0$

$(4;4)$ (Δ) (C)



(Δ) (C) **3**

u_3 u_2 u_1 u_0 **1 II**

(u_n) **2**

u_3 u_2 u_1 u_0

4 (u_n)

3

$0 \le u_n$ 4 **n**

P(0) •

$u_0 = 0$ $0 \le u_0$ 4 n=0

$0 \le u_n$ 4 P(n)

$0 \le u_{n+1}$ 4 P(n+1)

$\sqrt{8} \le u_{n+1}$ 4 **f** $f(0) \le f(u_n)$ $f(4)$ $0 \le u_n$ 4

$0 \le u_n$ 4 $\sqrt{8}$ 0 $0 \le u_{n+1}$ 4

(u_n)

$u_{n+1} - u_n$ (u_n)

$$u_{n+1} - u_n = \sqrt{2u_n + 8} - u_n = \frac{(\sqrt{2u_n + 8} - u_n)(\sqrt{2u_n + 8} + u_n)}{\sqrt{2u_n + 8} + u_n} = \frac{-u^2_n + 2u_n + 8}{\sqrt{2u_n + 8} + u_n}$$

$-u^2_n + 2u_n + 8$

$0 \le u_n$ 4 $u_n = -2$ $u_n = 4$ $-u^2_n + 2u_n + 8 = 0$

$-u^2_n + 2u_n + 8$

u_n	0	4	$+\infty$
		0	
	(u_n)		(u_n)

(u_n) $0 \le u_n$ 4

$4 - u_{n+1} \le \frac{1}{2}(4 - u_n)$ **n**

$$4 - u_{n+1} = 4 - \sqrt{2u_n + 8} = \frac{(4 - \sqrt{2u_n + 8})(4 + \sqrt{2u_n + 8})}{4 + \sqrt{2u_n + 8}} = \frac{(4)^2 - 2u_n - 8}{4 + \sqrt{2u_n + 8}} = \frac{2(4 - u_n)}{4 + \sqrt{2u_n + 8}}$$

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$$\frac{1}{2}(4-u_n)\times\frac{4}{4+\sqrt{2u_n+8}} \qquad \frac{2(4-u_n)}{4+\sqrt{2u_n+8}}$$

$$\frac{1}{2}(4-u_n)\times\frac{4}{4+\sqrt{2u_n+8}}\le\frac{1}{2}(4-u_n) \qquad 0 \quad \frac{4}{4+\sqrt{2u_n+8}} \quad 1$$

n $\qquad 4-u_{n+1}\le\frac{1}{2}(4-u_n)$

$$4-u_n\le\frac{1}{2^n}(4-u_0) \quad \mathbf{n}$$

$$4-u_{n+1}\le\frac{1}{2}(4-u_n)$$

$4-u_2\le\frac{1}{2}(4-u_1)$ n= 1 $\qquad 4-u_1\le\frac{1}{2}(4-u_0)$ n=0

$4-u_n\le\frac{1}{2}(4-u_{n-1})$ n 1

$$4-u_n\le\left(\frac{1}{2}\right)^n(4-u_0)$$

$$4-u_n\le\frac{1}{2^n}(4-u_0) \ \text{n}$$

$\lim\limits_{x\to+\infty} u_n$

$0\le 4-u_n\le\frac{4}{2^n}$ $\qquad 4-u_n\ge 0$ $\qquad 4-u_n\le\frac{4}{2^n}$

$\lim\limits_{x\to+\infty} u_n=4$ $\qquad \lim\limits_{x\to+\infty}(4-u_n)=0$ $\qquad \lim\limits_{x\to+\infty}\frac{4}{2^n}=0$

2 2016 25

$\lim\limits_{x\to+\infty} f(x)$ **1I**

$f(x)=\frac{5x}{x+2}$ $[0;+\infty[$ f

$$\lim_{x\to+\infty} f(x)=\lim_{x\to+\infty}\frac{5x}{x}=5$$

f

$f'(x)$

$f'(x)=\frac{5(x+2)-1(5x)}{(x+2)^2}=\frac{10}{(x+2)^2}$ $\qquad [0;+\infty[$ f

f $\qquad f'(x)$ 0

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x	0		$+\infty$
$f'(x)$			
$f(x)$	0	↗	5

$f(x) \geq 0$ $[0;+\infty[$ **x** **2**

$f(x) \geq 0$ $0 \leq f(x) \leq 5$ $[0;+\infty[$ **x** f

$1 \leq u_n \leq 3$ **n** **II**

P(0) •

$u_0 = 1$ $1 \leq u_0 \leq 3$ n=0

$1 \leq u_{n+1} \leq 3$ P(n+1) $1 \leq u_n \leq 3$ P(n)

$\frac{5}{3} \leq u_{n+1} \leq 3$ **f** $f(1) \leq f(u_n)$ $f(3)$ $1 \leq u_n \leq 3$

$n \in$ $1 \leq u_n \leq 3$ $\frac{5}{3}$ 1 $1 \leq u_{n+1} \leq 3$

(u_n)

$u_{n+1} - u_n$ (u_n)

$$u_{n+1} - u_n = f(u_n) - u_n = \frac{5u_n}{u_n + 2} - u_n = \frac{-u_n^2 + 3u_n}{u_n + 2} = \frac{u_n(-u_n + 3)}{u_n + 2}$$

$u_n = 3$ $u_n = 0$ $u_{n+1} - u_n = 0$

u_n	$-\infty$	1	0	3	$+\infty$
			0	0	
	(u_n)		(u_n)		(u_n)

(u_n) $1 \leq u_n \leq 3$

3 (u_n)

v_0 $\frac{2}{5}$ (v_n) **2**

$n \in$ $v_{n+1} = \frac{2}{5}.v_n$ $\frac{2}{5}$ (v_n)

$$v_n = 1 - \frac{3}{u_n} = \frac{u_n - 3}{u_n}$$

$$v_{n+1}=1-\frac{3}{u_{n+1}}=1-\frac{3}{f(u_n)}=1-\frac{3(u_n+2)}{5u_n}=\frac{5u_n-3(u_n+2)}{5u_n}=\frac{2(u_n-3)}{5u_n}=\frac{2}{5}v_n$$

$$v_0=1-\frac{3}{u_0}=-2$$

n u_n **n** v_n

$$v_n=v_0.q^n=-2.\left(\frac{2}{5}\right)^n \quad n\in \qquad (v_n)$$

$$u_n=\frac{3}{1-v_n}=\frac{3}{1+2.\left(\frac{2}{5}\right)^n} \qquad u_n=\frac{3}{1-v_n} \qquad v_n=1-\frac{3}{u_n}$$

(u_n)

u_n (u_n)

$\left(\frac{2}{5}\right)$ 1 $\lim\limits_{x\to+\infty}\left(\frac{2}{5}\right)^n=0$ $\lim\limits_{x\to+\infty}u_n=\lim\limits_{x\to+\infty}\frac{3}{1+2.\left(\frac{2}{5}\right)^n}=3$

n S_n **3**

$$\frac{1}{u_n}=\frac{1-v_n}{3}=\frac{1}{3}-\frac{1}{3}v_n \qquad S_n=\frac{1}{u_0}+\frac{1}{u_1}+.....+\frac{1}{u_n}$$

$$S_n=(\frac{1}{3}-\frac{1}{3}v_0)+(\frac{1}{3}-\frac{1}{3}v_1)+.....+(\frac{1}{3}-\frac{1}{3}v_n)=\frac{1}{3}(n+1)-\frac{1}{3}(v_0+v_1+...+v_n)$$

$$S_n=\frac{1}{3}(n+1)-\frac{1}{3}(v_0+v_1+...+v_n)=\frac{1}{3}(n+1)-\frac{1}{3}v_0\left[\frac{1-q^{n+1}}{1-q}\right]=\frac{1}{3}(n+1)+\frac{10}{9}(1-\left(\frac{2}{5}\right)^{n+1})$$

1 2016 26

$I=[0;4]$ **f** **1**

$x\in I$ $f'(x)$ 0 $I=[0;4]$ f

$$f'(x)=\frac{13(9x+13)-9(13x)}{(9x+13)^2}=\frac{169}{(9x+13)^2} \quad 0$$

I $f'(x)$ $x\in I$

f $f(0)\le f(x)\le f(4)$ $0\le x\le 4$ $x\in I$

$\frac{52}{49}<4$ $0\le f(x)\le 4$ $f(4)=\frac{52}{49}$ $f(0)=0$ $0\le f(x)\le\frac{52}{49}$

$0\le u_n\le 4$ **n** **2**

P(0)

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$u_0 = 4$ $0 \le u_0 \le 4$ n=0

$0 \le u_n \le 4$ P(n)

$0 \le u_{n+1} \le 4$ P(n+1)

$0 \le u_{n+1} \le 4$ **f** $f(0) \le f(u_n)$ $f(4)$ $0 \le u_n \le 4$

$0 \le u_n \le 4$

(u_n)

$u_{n+1} - u_n$ (u_n)

$$u_{n+1} - u_n = \frac{13u_n}{9u_n + 13} - u_n = \frac{-9u_n^2}{9u_n + 13}$$

(u_n)

$u_n \ge 0$ 0 (u_n)

$u_n \ne 0$ n 3

P(0)

$u_0 = 4$ $u_0 \ne 0$ n=0

$u_{n+1} \ne 0$ P(n+1) $u_n \ne 0$ P(n)

$f(0) = 0$ $u_{n+1} \ne 0$ **f** $f(u_n) \ne f(0)$ $u_n \ne 0$

n $u_n \ne 0$

v_0 (v_n) **4**

r $v_{n+1} - v_n = r$ n (v_n)

$$v_{n+1} - v_n = \frac{13}{u_{n+1}} - \frac{13}{u_n} = 13\left(\frac{9u_n + 13}{13u_n}\right) - \frac{13}{u_n} = \frac{9u_n + 13}{u_n} - \frac{13}{u_n} = 9$$

$$v_0 = 2 + \frac{13}{u_0} = 2 + \frac{13}{4} = \frac{21}{4}$$ r=9 (v_n)

n v_n

$$v_n = \frac{21}{4} + 9n \qquad v_n = v_0 + nr$$ n

$\lim\limits_{x \to +\infty} u_n$ **n** $u_n = \frac{52}{36n + 13}$

$$u_n = \frac{13}{v_n - 2} \qquad v_n - 2 = \frac{13}{u_n} \qquad v_n = 2 + \frac{13}{u_n}$$

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$$u_n = \frac{13}{\frac{21}{4}+9n-2} = \frac{13(4)}{36n+13} = \frac{52}{36n+13} \qquad v_n = \frac{21}{4}+9n$$

$$\lim_{x\to+\infty}(18n) = +\infty \qquad \lim_{x\to+\infty} u_n = \lim_{x\to+\infty}\frac{52}{36n+13} = \frac{13}{18n} = 0$$

2 2016 27

v_0 **q** (v_n) **1**

q $v_{n+1} = v_n.q$ n (v_n)

$$v_{n+1} = \frac{u_{n+1}-1}{u_{n+1}+2} = \frac{\frac{2u_n+2}{u_n+3}-1}{\frac{2u_n+2}{u_n+3}+2} = \frac{u_n-1}{4(u_n+2)} = \frac{1}{4}v_n$$

$$v_0 = \frac{u_0-1}{u_0+2} = -\frac{1}{2} \qquad q = \frac{1}{4} \qquad (v_n)$$

n v_n **2**

$$v_n = -\frac{1}{2}.\left(\frac{1}{4}\right)^n \qquad v_n = v_0.q^n \qquad n$$

n u_n

$$u_n = -\frac{2v_n+1}{v_n-1} \qquad v_n.u_n + 2v_n = u_n - 1 \qquad v_n(u_n+2) = u_n - 1 \qquad v_n = \frac{u_n-1}{u_n+2}$$

$$u_n = \frac{\left(\frac{1}{4}\right)^n - 1}{-\frac{1}{2}.\left(\frac{1}{4}\right)^n - 1} \qquad u_n = -\frac{2v_n+1}{v_n-1} \qquad v_n = -\frac{1}{2}.\left(\frac{1}{4}\right)^n$$

$\lim u_n$

0 q 1 $\lim v_n = 0$ $\lim u_n = \lim -\frac{2v_n+1}{v_n-1} = 1$

S_n **3**

$$S_n = v_0 + v_1 + \ldots + +v_n = v_0\left[\frac{1-q^{n+1}}{1-q}\right] = -\frac{1}{2}\left[\frac{1-\left(\frac{1}{4}\right)^{n+1}}{1-\left(\frac{1}{4}\right)}\right] = -\frac{2}{3}\left[1-\left(\frac{1}{4}\right)^{n+1}\right]$$

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n $\frac{1}{u_n+2}=\frac{1}{3}(1-v_n)$

$\frac{1}{u_n+2}=\frac{1}{3}(1-v_n)$ $u_n+2=-\frac{2v_n+1}{v_n-1}+2=\frac{-3}{v_n-1}$ $u_n=-\frac{2v_n+1}{v_n-1}$

S'_n n

$$S'_n=\frac{1}{u_0+2}+\frac{1}{u_1+2}+....+\frac{1}{u_n+2}=\frac{1}{3}\left[(1-v_0)+(1-v_1)+....+(1-v_n)\right]$$

$$S'_n=\frac{1}{3}\left[(n+1)-(v_0+v_1+.....+v_n)\right]=\frac{1}{3}\left[(n+1)-S_n\right]$$

1 2015 28

u_3 u_2 u_1 (1

$u_0=e^2-1$ $u_{n+1}=(1+u_n)e^{-2}-1$

$u_1=(1+u_0)e^{-2}-1=(1+e^2-1)e^{-2}-1=0$

$u_2=(1+u_1)e^{-2}-1=(1+0)e^{-2}-1=e^{-2}-1$

$u_3=(1+u_2)e^{-2}-1=(1+e^{-2}-1)e^{-2}-1=e^{-4}-1$

$1+u_n$ 0 n 2

$1+u_0=e^2$ 0 $1+u_0$ 0 n=0

k $n=k$ $1+u_n$ 0

$n=k+1$ $1+u_n$ 0

$1+u_{k+1}$ 0 $n=k+1$

$1+u_{k+1}=(1+u_k)e^{-2}-1+1=(1+u_k)e^{-2}$

$(1+u_k)e^{-2}$ 0 e^{-2} 0 $1+u_k$ 0

n $1+u_n$ 0 $1+u_{k+1}$ 0

(u_n) 3

n $u_{n+1}-u_n$ 0 (u_n)

$u_{n+1}-u_n=(1+u_n)e^{-2}-1-u_n=(e^{-2}-1)(1+u_n)$

$u_{n+1}-u_n$ 0 $(e^{-2}-1)$ 0 $1+u_n$ 0

(u_n)

1 u_n -1 (u_n)

(v_n) 4

$n\in$ $v_{n+1}=q.v_n$ q (v_n)

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$v_{n+1}=3(1+u_{n+1})=3(1+(1+u_n)e^{-2}-1)=3(1+u_n)e^{-2}$ $v_n=3(1+u_n)$

$q=e^{-2}$ (v_n) $v_{n+1}=3(1+u_n)e^{-2}=e^{-2}.v_n$

$v_0=3(1+u_0)=3e^2$

$\lim_{x\to+\infty} u_n$ n u_n v_n

$v_n=3e^2.(e^{-2})^n=3e^{2-2n}$ $v_n=v_0.q^n$ (v_n)

$u_n=-1+\frac{1}{3}v_n=e^{2-2n}-1$ $\frac{1}{3}v_n=1+u_n$ $v_n=3(1+u_n)$

$\ln v_0+\ln v_1+...+\ln v_n=(n+1)(-n+2+\ln 3)$ $n\in$

$\ln v_0+\ln v_1+...+\ln v_n=\ln(v_0.v_1.....v_n)=\ln(v_0^{n+1}.q^{\frac{n}{2}(n+1)})$

$\ln v_0+\ln v_1+...+\ln v_n=\ln(v_0^{n+1}.q^{\frac{n}{2}(n+1)})=(n+1)\ln(v_0.q^{\frac{n}{2}})$

$\ln v_0+\ln v_1+...+\ln v_n=(n+1)\ln(3e^2.(e^{-2})^{\frac{n}{2}})=(n+1)\ln(3e^{2-n})$

$\ln v_0+\ln v_1+...+\ln v_n=(n+1)(-n+2+\ln 3)$

2 2015 29

$[0;+\infty[$ f I(1-I

$f(x)=\frac{4x+1}{x+1}$ $[0;+\infty[$ f

$[0;+\infty[$ f

$f'(x)=\frac{4(x+1)-(4x+1)}{(x+1)^2}=\frac{3}{(x+1)^2}$

$[0;+\infty[$ $f'(x)$ 0

(D) (C_f) (2

$f(x)-y$ 0 (D) (C_f)

$f(x)-y=\frac{4x+1}{x+1}-x=\frac{-x^2+3x+1}{x+1}$

$x+1$ 0 $-x^2+3x+1$

$x_2=\frac{3+\sqrt{13}}{2}$ $x_1=\frac{3-\sqrt{13}}{2}\notin[0;+\infty[$ $-x^2+3x+1=0$

$[0;+\infty[$ $-x^2+3x+1$

x	0	x_2	$+\infty$
		0	

(D) (C_f)

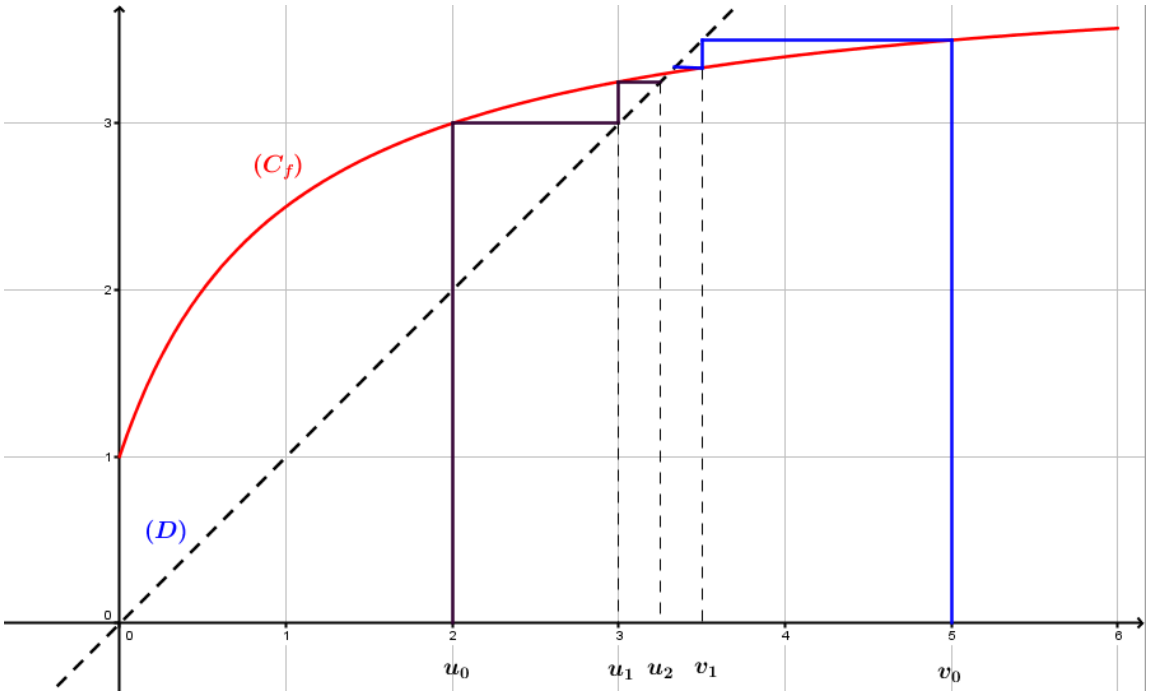
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(D) (C_f) $f(x)-y \quad 0$ $x \in [0;x_2[$

x_2 (D) (C_f) $f(x)-y=0$ $x=x_2$

(D) (C_f) $f(x)-y \quad 0$ $x \in]x_2;+\infty[$

$[0;6]$ (D) (C_f) (3



v_3 v_2 v_1 v_0 u_3 u_2 u_1 u_0 **1 II**

(v_n) (u_n)

(v_n) (u_n)

$$x_2=\frac{3+\sqrt{13}}{2}$$ (v_n) (u_n)

$x_2=\alpha$ $\alpha \le v_n \le 5$ $2 \le u_n \le \alpha$ **n** **2**

n $\alpha \le v_n \le 5$ $2 \le u_n \le \alpha$

P(0)

$\begin{cases} u_0=2 \\ v_0=5 \end{cases}$ $\begin{cases} 2 \le u_0 \le \alpha \\ \alpha \le v_0 \le 5 \end{cases}$ $n=0$

$\begin{cases} 2 \le u_n \le \alpha \\ \alpha \le v_n \le 5- \end{cases}$ P(n)

$\begin{cases} 2 \le u_{n+1} \le \alpha \\ \alpha \le v_{n+1} \le 5 \end{cases}$ P(n+1)

f $\begin{cases} f(2) \le f(u_n) \le f(\alpha) \\ f(\alpha) \le f(v_n) \le f(5) \end{cases}$ $\begin{cases} 2 \le u_n \le \alpha \\ \alpha \le v_n \le 5 \end{cases}$

$\begin{cases} 2 \quad 3 \le f(u_n) \le \alpha \\ \alpha \le f(v_n) \le \dfrac{7}{2} \quad 5 \end{cases}$ $\begin{cases} 3 \le f(u_n) \le f(\alpha) \\ f(\alpha) \le f(v_n) \le \dfrac{7}{2} \end{cases}$

n $\alpha \le v_n \le 5$ $2 \le u_n \le \alpha$

(v_n) (u_n)

(u_n)

2 I $u_{n+1} - u_n = \dfrac{-(u_n - x_1)(u_n - x_2)}{(u_n + 1)}$ $u_{n+1} - u_n = \dfrac{4u_n + 1}{u_n + 1} - u_n = \dfrac{-u_n^2 + 3u_n + 1}{u_n + 1}$:

u_n	0	2	α	$+\infty$
			0	

(u_n) $2 \le u_n \le \alpha$

(v_n)

$v_{n+1} - v_n = \dfrac{4v_n + 1}{v_n + 1} - v_n = \dfrac{-v_n^2 + 3v_n + 1}{v_n + 1}$:

2 I $v_{n+1} - v_n = \dfrac{-(v_n - x_1)(v_n - x_2)}{(v_n + 1)}$

v_n	0	α	5	$+\infty$
		0		

(v_n) $\alpha \le v_n \le 5$

$v_{n+1} - u_{n+1} \le \dfrac{1}{3}(v_n - u_n)$ $n \in$ **3**

$(v_{n+1} - u_{n+1}) - \dfrac{1}{3}(v_n - u_n) \quad 0$

$v_{n+1} - u_{n+1} = \dfrac{4v_n + 1}{v_n + 1} - \dfrac{4u_n + 1}{u_n + 1} = \dfrac{3(v_n - u_n)}{(v_n + 1)(u_n + 1)}$

$\dfrac{9}{(v_n + 1)(u_n + 1)} \quad 1$ $v_{n+1} - u_{n+1} = \dfrac{1}{3}\dfrac{(v_n - u_n)9}{(v_n + 1)(u_n + 1)} \le \dfrac{1}{3}(v_n - u_n)$

$0 \quad (v_n - u_n) \le \left(\frac{1}{3}\right)^{n-1} \quad n \in$

$(v_n - u_n) \quad 0$.....(1)

$$(v_n - u_n) \le \left(\frac{1}{3}\right)^{n-1}$$

$$v_{n+1} - u_{n+1} \le \frac{1}{3}(v_n - u_n)$$

$$v_1 - u_1 \le \frac{1}{3}(v_0 - u_0) \qquad n = 0$$

$$v_2 - u_2 \le \frac{1}{3}(v_1 - u_1) \qquad n = 1$$

$$v_n - u_n \le \frac{1}{3}(v_{n-1} - u_{n-1}) \qquad n - 1$$

$$(v_n - u_n) \le \left(\frac{1}{3}\right)^n (v_0 - u_0) = \left(\frac{1}{3}\right)^n .3$$

$0 \quad (v_n - u_n) \le \left(\frac{1}{3}\right)^{n-1}$ (2) (1) $(v_n - u_n) \le \left(\frac{1}{3}\right)^{n-1}$ ....(2)

$(v_n) \quad (u_n)$ $\lim_{x \to +\infty}(v_n - u_n) = 0$

$\lim_{x \to +\infty}(v_n - u_n) = 0$ $\lim_{x \to +\infty}\left(\frac{1}{3}\right)^{n-1} = 0 \quad 0 \quad (v_n - u_n) \le \left(\frac{1}{3}\right)^{n-1}$

$2 \le u_n \le \alpha \quad \alpha$ (u_n)

$f(L) = L \qquad L$ (u_n)

$\lim_{x \to +\infty}(v_n) = \alpha$ $\lim_{x \to +\infty}(u_n) = \alpha$ $f(\alpha) = \alpha$

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(v_n) **1**

$n \in$ $v_{n+1} = q.v_n \qquad q \qquad (v_n)$

$v_{n+1} = \frac{2}{3}u_n - \frac{4}{3} + 4 = \frac{2}{3}u_n + \frac{8}{3} = \frac{2}{3}(u_n + 4) = \frac{2}{3}v_n$ $v_{n+1} = u_{n+1} + 4$ $v_n = u_n + 4$

$v_0 = u_0 + 4 = 5$ $q = \frac{2}{3}$ (v_n)

$n \qquad u_n \quad v_n$ **2**

$v_0 = 5$ $q = \frac{2}{3}$ (v_n)

$v_n = 5.\left(\frac{2}{3}\right)^n$ $v_n = v_0.q^n$ v_n

$u_n = v_n - 4 = 5.\left(\frac{2}{3}\right)^n - 4$ $v_n = u_n + 4$

(u_n) **3**

$u_{n+1} - u_n$ (u_n)

$$u_{n+1} - u_n = \left(5.\left(\frac{2}{3}\right)^{n+1} - 4\right) - \left(5.\left(\frac{2}{3}\right)^n - 4\right) = 5.\left(\frac{2}{3}\right)^n \left(\left(\frac{2}{3}\right) - 1\right) = -\frac{5}{3}.\left(\frac{2}{3}\right)^n$$

(u_n) $-\frac{5}{3}.\left(\frac{2}{3}\right)^n$ 0 n

n S_n **4**

$$S_n = u_0 + u_1 + u_2 + ... + u_n$$

$$S_n = (v_0 - 4) + (v_1 - 4) + ... + (v_n - 4) = (v_0 + v_1 + ... + v_n) + (-4)(n+1) \qquad u_n = v_n - 4$$

$$S_n = 15(1 - \left(\frac{2}{3}\right)^{n+1}) + (-4)(n+1) \qquad (v_0 + v_1 + ... + v_n) = v_0\left[\frac{1 - q^{n+1}}{1 - q}\right] = 15(1 - \left(\frac{2}{3}\right)^{n+1})$$

(w_n) **5**

n $w_{n+1} - w_n$ 0 (w_n)

$q = \frac{2}{3}$ 1 v_0 0 (v_n) $w_{n+1} - w_n = \frac{5}{v_{n+1} + 5} - \frac{5}{v_n + 5} = 5\left[\frac{v_n}{v_{n+1} + 5} - \frac{v_{n+1}}{v_n + 5}\right]$ 0

$\lim_{x \to +\infty}(u_n - w_n)$

$\lim_{x \to +\infty}(u_n - w_n) = 0$ $\lim_{x \to +\infty}\left(\frac{2}{3}\right)^n = 0$ $\lim_{x \to +\infty}(w_n) = \lim_{x \to +\infty}(\frac{5}{v_n + 5} - 5) = -4$ $\lim_{x \to +\infty}(u_n) = \lim_{x \to +\infty}(5.\left(\frac{2}{3}\right)^n - 4) = -4$

2 2014 31

(u_n) **1- I**

$n \in$ $u_{n+1} = u.v_n$ q (u_n)

$u_{n+1} = e^{\frac{1}{2}-(n+1)} = e^{\frac{1}{2}-n}.e^{-1} = u_n.e^{-1}$ $u_{n+1} = e^{\frac{1}{2}-(n+1)}$ $u_n = e^{\frac{1}{2}-n}$

$u_0 = e^{\frac{1}{2}-0} = \sqrt{e}$ $q = e^{-1} = \frac{1}{e}$ (u_n)

$\lim_{x \to +\infty} u_n$ **2**

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$\lim_{x\to+\infty} e^{-n} = 0$ $\lim_{x\to+\infty} u_n = \lim_{x\to+\infty} e^{\frac{1}{2}-n} = 0$ $u_n = e^{\frac{1}{2}-n}$

n S_n **3**

$S_n = \sqrt{e}\left[\frac{1-(e^{-1})^{n+1}}{1-(e^{-1})}\right]$ $S_n = u_0\left[\frac{1-q^{n+1}}{1-q}\right]$ $S_n = u_0 + u_1 + u_2 + ... + u_n$

(v_n) n v_n **1** II

$v_n = \ln(e^{\frac{1}{2}-n}) = (\frac{1}{2}-n)\ln e = (\frac{1}{2}-n)$ $u_n = e^{\frac{1}{2}-n}$ $v_n = \ln(u_n)$

$v_{n+1} - v_n = \ln(u_{n+1}) - \ln(u_n) = \ln\frac{u_{n+1}}{u_n} = \ln e^{-1} = -1$ $v_n = \ln(u_n)$

$v_0 = \ln(u_0) = \ln\sqrt{e} = \frac{1}{2}$ (-1) (v_n)

n P_n **2**

$P_n = v_0 + v_1 + ... + v_n$ $P_n = \ln(u_0) + \ln(u_1) + ... + \ln(u_n)$ $P_n = \ln(u_0 \times u_1 \times ..... \times u_n)$

$P_n = \frac{(n+1)}{2}\left(\frac{1}{2}+\frac{1}{2}-n\right) = \frac{-n^2+1}{2}$ $P_n = \frac{(n+1)}{2}(v_0 + v_n)$

$P_n + 4n$ 0 **n**

$-n^2 + 8n + 1$ 0 $P_n + 4n$ 0 $P_n + 4n = \frac{-n^2+1+8n}{2}$

$n \in \{0;1;2;3;4;5;6;7;8\}$ $n \in$ $0 \le n \le 8$ $4-2\sqrt{5}$ n $4+2\sqrt{5}$

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(v_n) **1** I

$v_{n+1} = q.v_n : n \in$ (v_n)

$v_{n+1} = \frac{5}{6}v_n$ $v_{n+1} = \frac{5^{n+2}}{6^{n+1}} = \frac{5\times5^{n+1}}{6\times6^n} = \frac{5}{6}\times\frac{5^{n+1}}{6^n} = \frac{5}{6}\times v_n$

$v_0 = \frac{5^{0+1}}{6^0} = 5$ $\frac{5}{6}$ (v_n)

$\lim_{n\to+\infty} v_n$ **2**

-1 $\frac{5}{6}$ 1 $\lim_{n\to+\infty} v_n = \lim_{n\to+\infty} \frac{5^{n+1}}{6^n} = \lim_{n\to+\infty} 5.\left(\frac{5}{6}\right)^n = 0$

$n \in$ $1 \le u_n \le 6$ **1** II

$u_0 = 1$ $1 \le u_0 \le 6$ $n = 0$

$1 \le u_{n+1} \le 6$ $n \in$ $1 \le u_n \le 6$ $n \in$

$\sqrt{11} \le \sqrt{5u_n+6} \le 6$ $11 \le 5u_n+6 \le 36$ $5 \le 5u_n \le 30$ $1 \le u_n \le 6$

$1 \le u_{n+1} \le 6$ 1 $\sqrt{11} \le \sqrt{5u_n+6} \le 6$

(u_n) **2**

$u_{n+1}-u_n = \sqrt{5u_n+6}-u_n$

$$u_{n+1}-u_n = \frac{(\sqrt{5u_n+6}-u_n)(\sqrt{5u_n+6}+u_n)}{\sqrt{5u_n+6}+u_n} = \frac{-u_n^2+5u_n+6}{\sqrt{5u_n+6}+u_n} = \frac{-(u_n+1)(u_n-6)}{\sqrt{5u_n+6}+u_n}$$

$1 \le u_n \le 6$

$n \in$ $6-u_{n+1} \le \frac{5}{6}(6-u_n)$ **3**

$6-u_{n+1} \le \frac{5}{6}(6-u_n)$ $\frac{1}{6+\sqrt{5u_n+6}} \le \frac{1}{6}$ $6-u_{n+1} = 6-\sqrt{5u_n+6} = \frac{5(6-u_n)}{6+\sqrt{5u_n+6}} \le \frac{5}{6}(6-u_n)$

$\lim\limits_{n\to+\infty} u_n$ $n \in$ $0 \le 6-u_n \le v_n$

$n \in$ $6-u_{n+1} \le \frac{5}{6}(6-u_n)$

$6-u_n \le \frac{5}{6}(6-u_{n-1})$ n $6-u_2 \le \frac{5}{6}(6-u_1)$ 1 $6-u_1 \le \frac{5}{6}(6-u_0)$ 0

$0 \le 6-u_n \le 5\left(\frac{5}{6}\right)^n = v_n$ $6-u_n \le \left(\frac{5}{6}\right)^n (6-u_0)$

$\lim\limits_{n\to+\infty} u_n = 6$ $\lim\limits_{n\to+\infty}(6-u_n) = 0$ $\lim\limits_{n\to+\infty} v_n = 0$ $0 \le 6-u_n \le v_n$

2 2013 33

u_3 u_2 u_1 u_0 **1**

(u_n)

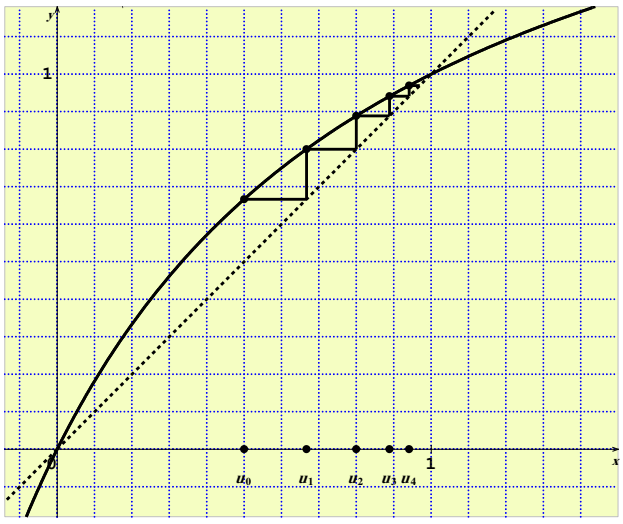
(u_n)

$[0;1]$ **f** 2

$f'(x)$ 0 $[0;1]$ f

$$f'(x) = \frac{2(x+1)-1(2x)}{(x+1)^2} = \frac{2}{(x+1)^2} \quad 0$$

0 u_n 1



$u_0 = \frac{1}{2}$ 0 u_0 1 $n=0$

0 u_n 1 n

$0 \quad u_{n+1} \quad 1 \quad n$

$f \qquad f(0) \quad f(u_n) \quad f(1) \qquad 0 \quad u_n \quad 1 \quad n$

$f(1)=1 \quad f(0)=0 \qquad 0 \quad u_{n+1} \quad 1$

(u_n)

$u_{n+1}-u_n \qquad (u_n)$

$$u_{n+1}-u_n=f(u_n)-u_n=\frac{2u_n}{u_n+1}-u_n=\frac{-u_n^2+u_n}{u_n+1}=\frac{u_n(1-u_n)}{u_n+1}$$

$(u_n) \qquad u_n(1-u_n) \quad 0 \qquad u_n \in]0;1[$

$\frac{1}{2} \qquad (v_n)$ **3**

$$v_{n+1}=\frac{1}{2}v_n : n \in \qquad \frac{1}{2} \qquad (v_n)$$

$$v_0=\frac{u_0-1}{u_0}=-1 \qquad v_{n+1}=\frac{1}{2}v_n \qquad v_{n+1}=\frac{u_{n+1}-1}{u_{n+1}}=\frac{f(u_n)-1}{f(u_n)}=\frac{u_n-1}{2u_n}=\frac{1}{2}.\frac{u_n-1}{u_n}$$

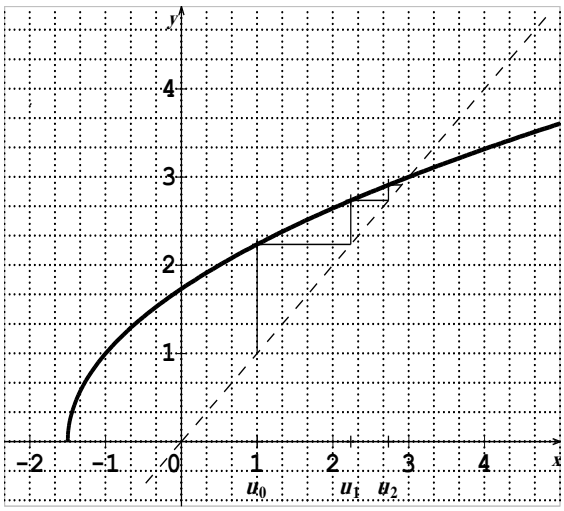
(u_n)

$$v_n=v_0.q^n=-1.\left(\frac{1}{2}\right)^n \qquad u_n=\frac{1}{1-v_n} \qquad v_n=\frac{u_n-1}{u_n}$$

$$\lim_{n\to+\infty} u_n=1 \qquad \lim_{n\to+\infty} u_n=\lim_{n\to+\infty}\frac{1}{1+2^{-n}}=1 \qquad u_n=\frac{1}{1+2^{-n}}$$

1 2012 34

$u_3 \quad u_2 \quad u_1 \quad u_0$ **1**



(u_n)

3 (u_n) 1

$0 \quad u_n \quad 3 \quad n$ **2**

$u_0=1 \qquad 0 \quad u_0 \quad 3 \qquad n=0$

$0 \quad u_n \quad 3 \quad n$

$0 \quad u_{n+1} \quad 3 \quad n$

$0 \quad u_n \quad 3 \quad n$

$h \qquad h(0) \quad h(u_n) \quad h(3)$

$\sqrt{3} \quad u_{n+1} \quad 3$

(u_n) **3**

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$u_{n\ 1}$ u_n (u_n)

$u_{n+1} - u_n = \sqrt{2u_n + 3} - u_n$ $\dfrac{u_n^2 \quad 2u_n + 3}{\sqrt{2u_n + 3} + u_n}$

$0 \quad u_n \quad 3$ $u_n^2 \quad 2u_n + 3 \quad 0$ $u_n = 3 \quad u_n = -1$ $u_n^2 \quad 2u_n + 3 = 0$

(u_n)

$u_1 \quad u_0$ h $\underline{\qquad 2 \qquad}$

(u_n)

$\lim\limits_{x} u_n$ (u_n)

$u_n \quad 3 \ 3$ (u_n)

$h(\alpha) = \alpha$ $\lim\limits_{x \to +\infty} u_n = \alpha$

α $2\alpha + 3 = \alpha^2$ $\sqrt{2\alpha + 3} = \alpha$ $h(\alpha) = \alpha$

(u_n) $\alpha = -1 \quad \alpha = 3$

2 2012 35

$3 \quad u_n \quad 4 \ n$ **1**

$u_0 = \dfrac{13}{4}$ $3 \quad u_0 \quad 4$ $n = 0$

$3 \quad u_n \quad 4 \ n$

$3 \quad u_{n+1} \quad 4 \ n$

$0 \quad \sqrt{u_n - 3} \quad 1 \quad 0 \quad u_n - 3 \quad 1$ $3 \quad u_n \quad 4 \ n$

$3 \quad u_{n+1} \quad 4$ $3 \quad 3 + \sqrt{u_n - 3} \quad 4$

$u_{n+1} - u_n = \dfrac{-u_n^2 + 7u_n - 12}{\sqrt{u_n - 3} + u_n - 3}$ n **2**

$\sqrt{u_n - 3} - (3 - u_n)$ $u_{n+1} - u_n = \sqrt{u_n - 3} + 3 - u_n = \dfrac{-u_n^2 + 7u_n - 12}{\sqrt{u_n - 3} + u_n - 3}$

(u_n)

$-u_n^2 + 7u_n - 12$ $u_{n+1} - u_n$

$-u_n^2 + 7u_n - 12 \quad 0$ $3 \quad u_n \quad 4$ $4 \quad 3$ $-u_n^2 + 7u_n - 12$

(u_n)

3

4 4 (u_n)

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$\frac{1}{2}$ (v_n) **4**

$v_{n+1}=\frac{1}{2}v_n$ $\frac{1}{2}$ (v_n)

$v_0=\ln(u_0-3)=\ln\frac{1}{4}$ $v_{n+1}=\ln(u_{n+1}-3)=\ln(\sqrt{u_n-3})=\frac{1}{2}v_n$

$\lim u_n$ **n** u_n v_n

$v_n=v_0.q^n=-\ln 4\left(\frac{1}{2}\right)^n$

$u_n=3+e^{v_n}=3+e^{-\ln 4.\left(\frac{1}{2}\right)^n}$ $e^{v_n}=u_n-3$ $v_n=\ln(u_n-3)$

$\lim u_n=\lim(3+e^{-2\ln 2.\left(\frac{1}{2}\right)^n})=3+1=4$

$\lim P_n=\frac{1}{16}$ **n** P_n

$P_n=e^{v_0}.e^{v_1}.....e^{v_n}=e^{v_0+v_1+...+v_n}=e^{v_0\frac{1-(q)^{n+1}}{1-(q)}}$ $P_n=(u_0-3)(u_1-3)\times.....\times(u_n-3)$

$\lim P_n=\lim e^{\ln\frac{1}{16}\left(1-\left(\frac{1}{2}\right)^{n+1}\right)}=e^{\ln\frac{1}{16}}=\frac{1}{16}$ $P_n=e^{\ln\frac{1}{16}\left(1-\left(\frac{1}{2}\right)^{n+1}\right)}$ $\ln P_n=\ln\frac{1}{16}(1-\left(\frac{1}{2}\right)^{n+1})$

1 2011 36

$v_{n+1}=u_{n+1}+\frac{1}{2}=3(u_n+\frac{1}{2})=3v_n$		1
$\lim_{x\to+\infty} u_n=\lim_{x\to+\infty}(v_n-\frac{1}{2})=\lim_{x\to+\infty}(-\frac{1}{2}.3^n-\frac{1}{2})=-\infty$	$-\infty$	2
$S_n=-\frac{1}{2}\left[1+3+3^2+....+3^n\right]=-\frac{1}{2}\left[1.\frac{1-3^{n+1}}{1-3}\right]=\frac{1-3^{n+1}}{4}$	S_n	3

2 2011 37

α (v_n) **1**

$n\in$ $v_{n+1}=\alpha.v_n$ α (v_n)

$u_{n+1}=\alpha u_n+1$ $v_n=u_n+\frac{1}{\alpha-1}$

$$v_{n+1}=u_{n+1}+\frac{1}{\alpha-1}=\alpha u_n+1+\frac{1}{\alpha-1}=\alpha(u_n+\frac{1}{\alpha-1})=\alpha v_n$$

α **n** u_n α **n** v_n

$$v_n=\left(\frac{6\alpha-5}{\alpha-1}\right).\alpha^n \qquad v_0=u_0+\frac{1}{\alpha-1}=6+\frac{1}{\alpha-1} \qquad v_n=v_0.q^n$$

$$u_n=\left(\frac{6\alpha-5}{\alpha-1}\right).\alpha^n-\frac{1}{\alpha-1} \qquad u_n=v_n-\frac{1}{\alpha-1} \qquad v_n=u_n+\frac{1}{\alpha-1}$$

(u_n) α

$0 \quad \alpha \quad 1$ (u_n)

T_n S_n **2**

$$S_n=v_0(\frac{\alpha^{n+1}-1}{\alpha-1})=8\left[\frac{\left(\frac{3}{2}\right)^{n+1}-1}{\frac{3}{2}-1}\right]=16\left(\left(\frac{3}{2}\right)^{n+1}-1\right)$$

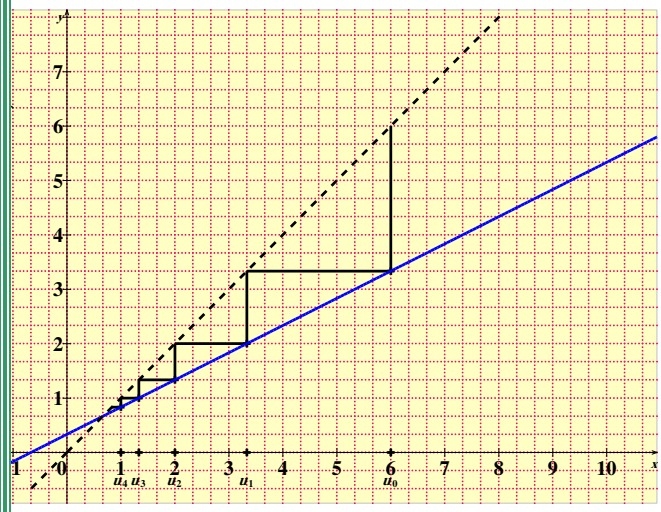
$$u_n=v_n-2 \qquad u_n=v_n-\frac{1}{\alpha-1} \qquad T_n=u_0+u_1+.....+u_n$$

$$T_n=(v_0-2)+(v_1-2)+.....+(v_n-2)$$

$$=(v_0+v_1+.....+v_n)-2(n+1)=S_n-2(n+1)$$

1 2010 38

u_4 u_3 u_2 u_1 u_0 **1**



(D) (Δ)

(D) (Δ)

$$\begin{cases} x=\frac{2}{3} \\ y=x \end{cases} \qquad \begin{cases} x=\frac{1}{2}x+\frac{1}{3} \\ y=x \end{cases} \qquad \begin{cases} y=\frac{1}{2}x+\frac{1}{3} \\ y=x \end{cases}$$

$(\frac{2}{3};\frac{2}{3})$ (D) (Δ)

(u_n)

(u_n)

u_n $\frac{2}{3}$ $n\in$ **2**

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$u_0 = 6$ u_0 $\frac{2}{3}$ $n = 0$

u_{n+1} $\frac{2}{3}$ $n \in$ u_n $\frac{2}{3}$ $n \in$

u_{n+1} $\frac{2}{3}$ $\frac{1}{2}u_n + \frac{1}{3}$ $\frac{1}{3} + \frac{1}{3}$ $\frac{1}{2}u_n$ $\frac{2}{3}\left(\frac{1}{2}\right)$ u_n $\frac{2}{3}$

(u_n)

u_n $\frac{2}{3}$ $u_{n+1} - u_n$ 0 $u_{n+1} - u_n = \frac{1}{2}u_n + \frac{1}{3} - u_n = -\frac{1}{2}(u_n - \frac{2}{3})$

(v_n) **3**

$v_{n+1} = q.v_n : n \in$ (v_n)

$v_{n+1} = u_{n+1} - \frac{2}{3} = \frac{1}{2}u_n + \frac{1}{3} - \frac{2}{3} = \frac{1}{2}(u_n - \frac{2}{3})$ $v_n = u_n - \frac{2}{3}$

$v_0 = u_0 - \frac{2}{3} = 6 - \frac{2}{3} = \frac{16}{3}$ $\frac{1}{2}$ (v_n) $v_{n+1} = \frac{1}{2}v_n$

n u_n v_n

$u_n = v_n + \frac{2}{3} = \frac{16}{3}.\left(\frac{1}{2}\right)^n + \frac{2}{3}$ $v_n = u_n - \frac{2}{3}$ $v_n = \frac{16}{3}.\left(\frac{1}{2}\right)^n$ $v_n = v_0.q^n$

n S'_n S_n

$S_n = \frac{16}{3}\left[\frac{1-\left(\frac{1}{2}\right)^{n+1}}{1-\frac{1}{2}}\right] = \frac{32}{3}\left[1-\left(\frac{1}{2}\right)^{n+1}\right]$ $S_n = v_0 + v_1 + ... + v_n = v_0\left[\frac{1-q^{n+1}}{1-q}\right]$

$S'_n = u_0 + u_1 + ... + u_n = (\frac{2}{3} + v_0) + (\frac{2}{3} + v_1) + ... + (\frac{2}{3} + v_n) = \frac{2}{3}(n+1) + S_n$

1 **2009** **39**

v_1 v_0 **1**

$v_1 = u_2 - u_1 = \frac{1}{3}$ $v_0 = u_1 - u_0 = 1$ $v_n = u_{n+1} - u_n$

(v_n) **2**

$v_{n+1} = v_n \times q$ (v_n)

$v_{n+1} = u_{n+2} - u_{n+1} = \frac{4}{3}u_{n+1} - \frac{1}{3}u_n - u_{n+1}$ $v_n = u_{n+1} - u_n$

$\boxed{q=\frac{1}{3}}$ (v_n) $v_{n+1}=u_{n+2}-u_{n+1}=\frac{1}{3}(u_{n+1}-u_n)=\frac{1}{3}v_n$

n S_n **3**

$$S_n=v_0\left[\frac{1-q^n}{1-q}\right]=1\left[\frac{1-\left(\frac{1}{3}\right)^n}{1-\left(\frac{1}{3}\right)}\right]=\frac{3}{2}\left(1-\left(\frac{1}{3}\right)^n\right) \qquad S_n=v_0+v_1+.....+v_{n-1}$$

$n\in$ $u_n=\frac{3}{2}\left(1-\left(\frac{1}{3}\right)^n\right)+1$

$S_n=(u_1-u_0)+(u_2-u_1)+.....+(u_n-u_{n-1})$ $S_n=v_0+v_1+.....+v_{n-1}$

$\boxed{u_n=\frac{3}{2}\left(1-\left(\frac{1}{3}\right)^n\right)+1}$ $u_n=u_0+S_n$ $S_n=-u_0+u_n$

(u_n)

$\alpha\in$ $\lim\limits_{x\to+\infty}u_n=\alpha$ (u_n)

$\lim\limits_{x\to+\infty}\left(\frac{1}{3}\right)^n=0$ $\lim\limits_{x\to+\infty}u_n=\lim\limits_{x\to+\infty}\frac{3}{2}\left(1-\left(\frac{1}{3}\right)^n\right)+1=\frac{5}{2}$

2 2009 40

q u_2 **1**

$\begin{cases}q=3\vee q=\frac{1}{3}\\u_2=6\end{cases}$ $\begin{cases}1+2q+q^2=\frac{16}{3}q\\u_2=6\end{cases}$ $\begin{cases}\frac{u_2}{q}+2u_2+u_2q=32\\u_2^3=216\end{cases}$ $\begin{cases}u_1+2u_2+u_3=32\\u_1\times u_2\times u_3=216\end{cases}$

$u_2=6$ $q=3$

$u_1=\frac{u_2}{q}=\frac{6}{3}=2$ $u_2=u_1q$

n u_n

$u_n=u_1q^{n-1}=2.3^{n-1}$

n S_n

$S_n=2\left(\frac{3^n-1}{3-1}\right)=3^n-1$ $S_n=u_1+u_2+....+u_{n-1}=u_1\left(\frac{q^n-1}{q-1}\right)$

$S_n=728$ **n**

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n=6 $3^n = 279 = 3^6$ $3^n - 1 = 278$ $S_n = 728$

v_3 v_2 **2**

$v_3 = \frac{3}{2}v_2 + u_2 = \frac{3}{2}(5) + 6 = \frac{27}{2}$ $v_2 = \frac{3}{2}v_1 + u_1 = 5$ $v_1 = 2$ $w_{n+1} = \frac{3}{2}v_n + u_n$

$\frac{1}{2}$ (w_n)

$w_{n+1} = \frac{1}{2}w_n$ $\frac{1}{2}$ (w_n)

$w_{n+1} = \frac{v_{n+1}}{u_{n+1}} - \frac{2}{3} = w_{n+1} = \frac{\frac{3}{2}v_n + u_n}{3u_n} - \frac{2}{3} = \frac{\frac{3}{2}v_n}{3u_n} + \frac{1}{3} - \frac{2}{3} = \frac{1}{2}\left(\frac{v_n}{u_n} - \frac{2}{3}\right) = \frac{1}{2}w_n$ $w_n = \frac{v_n}{u_n} - \frac{2}{3}$

n v_n **n** w_n

$w_n = \frac{1}{3}\left(\frac{1}{2}\right)^{n-1}$ $w_1 = \frac{v_1}{u_1} - \frac{2}{3} = \frac{1}{3}$ $w_n = w_1(q)^{n-1}$

$v_n = 2.3^{n-1}\left(\frac{1}{3}\left(\frac{1}{2}\right)^{n-1} + \frac{2}{3}\right)$ $v_n = u_n(w_n + \frac{2}{3})$ $w_n + \frac{2}{3} = \frac{v_n}{u_n}$ $w_n = \frac{v_n}{u_n} - \frac{2}{3}$

1 2008 41

I **f** **1**

I f $f'(x) = \frac{6}{(-x+4)^2}$ 0 *I* f

I **f(x)** ***I*** **x**

I f(x) $f\,f(1) \le f(x) \le f(2)$ $1 \le x \le 2$ I

$1 \le u_n \le 2$ **N** **n** **2**

$1 \le u_0 \le 2$ n=0

$1 \le u_{n+1} \le 2$ P(n+1) $1 \le u_n \le 2$ P(n)

2 $2 \le -u_n + 4 \le 3$ $1 \le u_n \le 2$ 1 $3 \le u_n + 2 \le 4$ $1 \le u_n \le 2$

n P(n) $1 \le \frac{u_n + 2}{-u_n + 4} \le 2$ 2 1

(u_n)

$u_{n+1} - u_n = \frac{u_n^2 - 4u_n + 2}{-u_n + 4} = \frac{(u_n - 1)(u_n - 2)}{-u_n + 4}$ $u_{n+1} - u_n = \frac{u_n + 2}{-u_n + 4} - u_n = \frac{u_n^2 - 4u_n + 2}{-u_n + 4}$

(u_n) 2 $u_n + 4$ 3 1 u_n 2

(u_n)

I (u_n)

$u_n = 1 + \dfrac{1}{\left(\frac{3}{2}\right)^n + 1}$ **N** **n** **3**

$u_0 = 1 + \dfrac{1}{\left(\frac{3}{2}\right)^0 + 1} = \dfrac{3}{2}$ n=0

$u_{n+1} = 1 + \dfrac{1}{\left(\frac{3}{2}\right)^{n+1} + 1}$ P(n+1) $u_n = 1 + \dfrac{1}{\left(\frac{3}{2}\right)^n + 1}$ P(n)

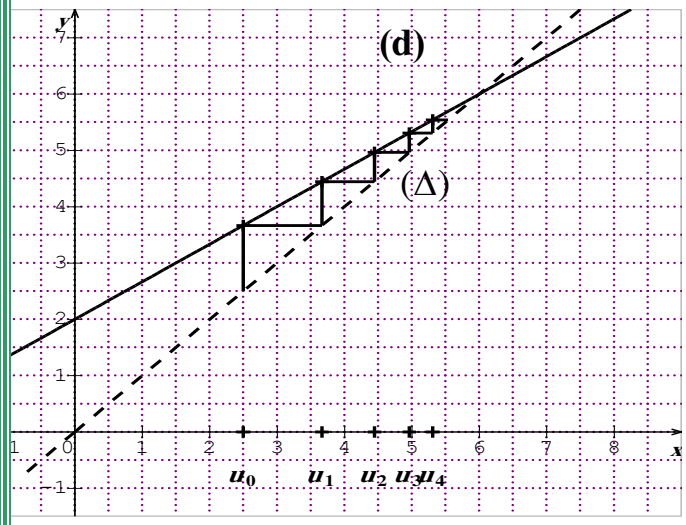
$$u_{n+1} = -1 + \frac{6}{4 - u_n} = \frac{3\left(\frac{3}{2}\right)^n + 4}{3\left(\frac{3}{2}\right)^n + 2} = 1 + \frac{1}{\left(\frac{3}{2}\right)^{n+1} + 1}$$

$\lim\limits_{n \to +\infty} u_n$

$\lim\limits_{n \to +\infty} u_n = 1$ $\lim\limits_{n \to +\infty} \dfrac{1}{\left(\frac{3}{2}\right)^{n+1} + 1} = 0$ $\lim\limits_{n \to +\infty} \left(\frac{3}{2}\right)^n = +\infty$

2 2009 42

u_4 u_3 u_2 u_1 u_0 **(d)** Δ **1**



u_n

u_n

6

$u_n \leq 6$ $n \in$ **2**

$u_0 = \dfrac{5}{2} \leq 6$ n=0

$u_n \leq 6$ P(n)

$u_{n+1} \leq 6$ P(n+1)

$\dfrac{2}{3}u_n + 2 \leq 6$ $\dfrac{2}{3}u_n \leq 4$ $u_n \leq 6$

N n $u_n \leq 6$ $u_{n+1} \leq 6$

u_n

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$u_{n+1}-u_n=\frac{2}{3}u_n+2-u_n=-\frac{1}{3}(u_n-6)$ 0 u_n

u_n

6 6 u_n

v_n **3**

$v_{n+1}=v_n.q$ v_n

$v_{n+1}=u_{n+1}-6=\frac{2}{3}u_n+2-6=\frac{2}{3}(u_n-6)$ $v_n=u_n-6$

$v_0=-\frac{7}{2}$ $\frac{2}{3}$ v_n $v_{n+1}=\frac{2}{3}v_n$

$\lim_{n\to+\infty} u_n$ **n** **u_n**

$\lim_{n\to+\infty} u_n=6$ $u_n=-\frac{7}{2}\left(\frac{2}{3}\right)^n+6$ $u_n=v_n+6$ $v_n=u_n-6$

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(v_n) **1**

$n \in$ $v_{n+1} = q.v_n$ q (v_n)

$v_{n+1} = u_{n+1} - 3(n+1) + 1 = u_{n+1} - 3n - 2$ $v_n = u_n - 3n + 1$ (v_n)

$u_0 = 0$ $u_{n+1} = 7u_n - 18n + 9$

$v_{n+1} = 7u_n - 18n + 9 - 3n - 2 = 7u_n - 21n + 7 = 7(u_n - 3n + 1) = 7v_n$

$v_0 = u_0 - 3(0) + 1 = 1$ 7 (v_n)

n u_n **n** v_n **2**

$v_n = 7^n$ $v_n = v_0.q^n$ n

$u_n = 7^n + 3n - 1$ $u_n = v_n + 3n - 1$ $v_n = u_n - 3n + 1$

n S_n **3**

$u_n = v_n + 3n - 1$ $S_n = u_0 + u_1 + ... + u_n$

$S_n = (v_0 + v_1 + ... + v_n) + (-1 + 2 + 5 + ... + (3n-1))$

$S_1 = v_0 + v_1 + ... + v_n$

$S_2 = (-1 + 2 + 5 + ... + (3n-1))$

$S_2 = \frac{n+1}{2}(-1 + (3n-1)) = \frac{(n+1)(3n-2)}{2}$ $S_1 = v_0\left(\frac{q^{n+1}-1}{q-1}\right) = \left(\frac{7^{n+1}-1}{6}\right)$

$S_n = S_1 + S_2 = \left(\frac{7^{n+1}-1}{6}\right) + \frac{(n+1)(3n-2)}{2}$

9 7^n **4**

$7^3 \equiv 1[9]$ $7^2 \equiv 4[9]$ $7^1 \equiv 7[9]$ $7^0 \equiv 1[9]$

3 9 7^n

$n =$	$3k$	$3k+1$	$3k+2$
$7^n \equiv$	1	7	4

9 $1440^{2019} + 1962^{1954} + 1954^{1962}$

$-7 \equiv 2[9]$ $1442^{2019} \equiv (-7)^{2019}[9]$ $1442^{2019} \equiv 2^{2019}[9]$ $1442 \equiv 2[9]$

$1442^{2019} \equiv 7^{2019}.(-1)^{2019}[9]$ $1442^{2019} \equiv (-7)^{2019}[9]$

$(-1)^{2019} \equiv -1[9]$ $7^{2019} = 7^{3(673)} \equiv 1[9]$

$1442^{2019} \equiv -1[9].....(1)$ $1442^{2019} \equiv 2^{2019}[9]$

$1962^{1954} \equiv 0[9].....(2)$ $1962 \equiv 0[9]$

$1954^{2062} \equiv 1[9].....(3)$ $1954 \equiv 1[9]$

0 $1440^{2019} + 1962^{1954} + 1954^{1962} \equiv 0[9]$ 3 2 1

$6S_n - 7u_n \equiv 0[9]$ **n**

$$6S_n = 6\left(\frac{7^{n+1}-1}{6}\right) + \frac{6(n+1)(3n-2)}{2} = 7^{n+1} + 9n^2 + 3n - 7$$

$$7u_n = 7^{n+1} + 21n - 7$$

$$6S_n - 7u_n = 9n^2 - 18n = 9(n^2 - 2n) \equiv 0[9]$$

1 2018 44

$u_n > \frac{1}{e}$ **n** **1**

(u_n) $f(x) = \frac{2x}{ex+1}$ $[0;+\infty[$ f

$u_{n+1} = f(u_n)$: n $u_0 = \frac{5}{4e}$

P(0) •

$u_0 = \frac{5}{4e} > \frac{1}{e}$ $u_0 > \frac{1}{e}$ n=0

$u_{n+1} > \frac{1}{e}$ P(n+1) $u_n > \frac{1}{e}$ P(n)

$f'(x) = \frac{2}{(ex+1)^2}$ 0 f $f(u_n) > f(\frac{1}{e})$ $u_n > \frac{1}{e}$

n ∈ $u_n > \frac{1}{e}$ $f(\frac{1}{e}) = \frac{1}{e}$ $f(u_n) = u_{n+1}$ $u_{n+1} > \frac{1}{e}$

$$u_{n+1} - u_n = \frac{e.u_n(\frac{1}{e} - u_n)}{e.u_n + 1}$$ n

$$u_{n+1} - u_n = f(u_n) - u_n = \frac{2u_n}{eu_n+1} - u_n = \frac{u_n - eu_n^2}{eu_n+1} = \frac{e.u_n(\frac{1}{e} - u_n)}{e.u_n+1}$$

(u_n)

$u_n > \frac{1}{e}$ $u_{n+1} - u_n$ $u_{n+1} - u_n = \frac{e.u_n(\frac{1}{e} - u_n)}{e.u_n + 1}$

(u_n)

$u_n > \frac{1}{e}$ (u_n)

n v_n v_0 **2** (v_n) **2**

n $v_{n+1} = 2v_n$ 2 (v_n)

$$v_{n+1} = \frac{e.u_{n+1}}{e.u_{n+1} - 1} = \frac{e.f(u_n)}{e.f(u_n) - 1} = \frac{e\frac{2u_n}{eu_n + 1}}{e\frac{2u_n}{eu_n + 1} - 1} = 2\frac{eu_n}{eu_n - 1} = 2v_n \qquad v_n = \frac{e.u_n}{e.u_n - 1}$$

$$u_0 = \frac{5}{4e} \qquad v_0 = \frac{e.u_0}{e.u_0 - 1} = 5$$

$v_n = 5 \times 2^n$ n $v_n = v_0 \times q^n$ (v_n)

$v_n = 1 + \frac{1}{e.u_n - 1}$ n **3**

$$v_n = 1 + \frac{1}{e.u_n - 1} = \frac{e.u_n - 1 + 1}{e.u_n - 1} = \frac{e.u_n}{e.u_n - 1} \qquad n$$

$\lim_{x \to +\infty} u_n$ u_n

$$u_n = \frac{1}{ev_n - e} + \frac{1}{e} \qquad e.u_n - 1 = \frac{1}{v_n - 1} \qquad v_n - 1 = \frac{1}{e.u_n - 1} \qquad v_n = 1 + \frac{1}{e.u_n - 1}$$

$$\lim_{x \to +\infty} v_n = 5 \lim_{x \to +\infty} 2^n = +\infty \qquad \lim_{x \to +\infty} u_n = \lim_{x \to +\infty} \frac{1}{ev_n - e} + \frac{1}{e} = \frac{1}{e}$$

S_n **n**

$$S_n = v_0\left[\frac{q^{n+1} - 1}{q - 1}\right] = 5\left[\frac{2^{n+1} - 1}{2 - 1}\right] = 5(2^{n+1} - 1) \qquad S_n = v_0 + v_1 + \ldots + v_n$$

7 2^n **n** **4**

$2^3 \equiv 1[7]$ $2^2 \equiv 4[7]$ $2^1 \equiv 2[7]$ $2^0 \equiv 1[7]$

3 7 2^n

$n =$	$3k$	$3k+1$	$3k+2$
$2^n \equiv$	1	2	4

7 S_n **n**

7 5 $(2^{n+1}-1)\equiv 0[7]$ $S_n=5(2^{n+1}-1)\equiv 0[7]$

$n+1=3k$ $2^{n+1}\equiv 1[7]$

k $n=3k+2$ k $n=3k-1$

2 2018 45

$\frac{5}{3}$ (w_n) **1**

$u_n=2(3)^n$ (u_n)

$v_{n+1}=5v_n+u_n$ n $v_0=4$ (v_n)

$w_n=\frac{v_n}{u_n}+\frac{1}{2}$ n

n $w_{n+1}=\frac{5}{3}w_n$ $\frac{5}{3}$ (w_n)

$w_{n+1}=\frac{v_{n+1}}{u_{n+1}}+\frac{1}{2}=\frac{5v_n+u_n}{3u_n}=\frac{5v_n}{3u_n}+\frac{1}{3}+\frac{1}{2}=\frac{5}{3}.\frac{v_n}{u_n}+\frac{5}{6}=\frac{5}{3}(\frac{v_n}{u_n}+\frac{1}{2})=\frac{5}{3}w_n$ $w_n=\frac{v_n}{u_n}+\frac{1}{2}$

$w_0=\frac{v_0}{u_0}+\frac{1}{2}=2+\frac{1}{2}=\frac{5}{2}$ $\frac{5}{3}$ (w_n)

$u_0=2(3)^0=2$ 3 (u_n)

n w_n **2**

$w_n=w_0q^n=\frac{5}{2}.\left(\frac{5}{3}\right)^n=\frac{5^{n+1}}{2.3^n}$ $w_0=\frac{5}{2}$ $\frac{5}{3}$ (w_n)

$v_n=5^{n+1}-3^n$ **n**

$v_n=\frac{5^{n+1}}{2.3^n}.2.3^n-\frac{2.3^n}{2}=5^{n+1}-3^n$ $w_n.u_n-\frac{u_n}{2}=v_n$ $w_n=\frac{v_n}{u_n}+\frac{1}{2}$

8 5^n 3^n **n** **3**

8 3^n

8 3^n $3^2\equiv 1[8]$ $3^1\equiv 3[8]$ $3^0\equiv 1[8]$

2

$n=$	$2k$	$2k+1$	
$3^n\equiv$	1	3	$[8]$

8 5^n

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2 8 5^n $5^2 \equiv 1[8]$ $5^1 \equiv 5[8]$ $5^0 \equiv 1[8]$

$n =$	$2k$	$2k+1$	
$5^n \equiv$	1	5	[8]

8 v_n **n** **4**

8 v_n $v_n = 5^{n+1} - 3^n$

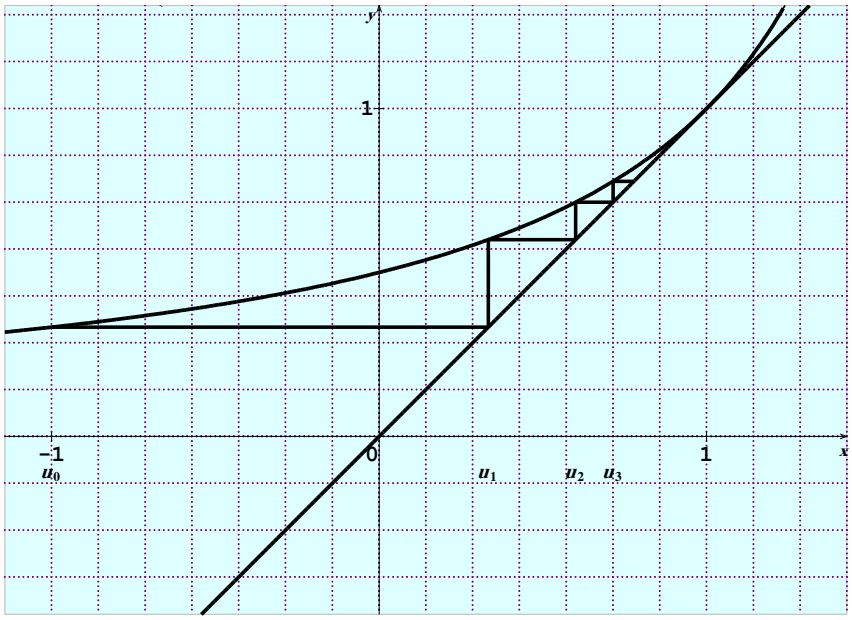
$n =$	$2k$	$2k+1$	
$5^n \equiv$	1	5	[8]
$5^{n+1} \equiv$	5	1	[8]
$3^n \equiv$	1	3	[8]
$5^{n+1} - 3^n \equiv$	4	6	[8]

4 8 v_n $n = 2k$

6 8 v_n $n = 2k+1$

1 2017 46

u_3 u_2 u_1 u_0 1



(u_n)

1 (u_n)

$u_n \leq 1$ **n** **2**

P(0) •

$u_0 = -1$ u_0 1 n=0

u_{n+1} 1 P(n+1) u_n 1 P(n)

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$f'(x)=\frac{1}{(2-x)^2}$ 0 **f** $f(u_n)$ $f(1)$ $u_n \le 1$

$f(1)=1$ $f(u_n)=u_{n+1}$ u_{n+1} 1

$n \in$ $u_n \le 1$

3 (u_n)

$u_{n+1}-u_n$ (u_n)

$$u_{n+1}-u_n=\frac{1}{2-u_{n+1}}-u_n=\frac{2-u_n}{-2u_n+3}-u_n=\frac{2(u_n-1)^2}{-2u_n+3}$$

u_n 1

(u_n)

1 (u_n)

2 (v_n) **4**

$n \in$ $v_{n+1}-v_n=2$ 2 (v_n)

$$v_{n+1}=\frac{1}{2-u_{n+1}}=\frac{2(2-u_n)}{1-u_n} \qquad v_n=\frac{2}{1-u_n}$$

$$v_{n+1}-v_n=\frac{2(2-u_n)}{1-u_n}-\frac{2}{1-u_n}=\frac{2(1-u_n)}{1-u_n}=2$$

n v_n

$v_n=v_0+nr$ n

$v_n=2n+1$ $r=2$ $v_0=\frac{2}{1-u_0}=1$

$\lim\limits_{x\to+\infty} u_n$ n u_n

$u_n=1-\frac{2}{2n+1}=\frac{2n-1}{2n+1}$ $u_n=1-\frac{2}{v_n}$ $1-u_n=\frac{2}{v_n}$ $v_n=\frac{2}{1-u_n}$

$\lim\limits_{x\to+\infty} v_n=+\infty$ $\lim\limits_{x\to+\infty} u_n=\lim\limits_{x\to+\infty} 1-\frac{2}{v_n}=1$

2 2017 47

u_n 0 n **1**

$u_{n+1}=\frac{n+1}{\alpha n}u_n$ n $u_1=\frac{1}{\alpha}$ (u_n)

u_n 0 n

P(1)

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2 α $u_1 = \frac{1}{\alpha}$ u_1 0 n=1

u_{n+1} 0 P(n+1) u_n 0 P(n)

2 αu_n 0 1 $(n+1)u_n$ 0 u_n 0

u_{n+1} 0 $\frac{n+1}{\alpha n}u_n$ 0 2 1

n u_n 0

(u_n)

n $u_{n+1} - u_n$ 0 (u_n)

$$u_{n+1} - u_n = \frac{n+1}{\alpha n}u_n - u_n = u_n(\frac{(1-\alpha)n+1}{\alpha n})$$

$(1-\alpha)n+1$

$(1-\alpha)n+1 \le 1-n$ 0 $(1-\alpha)n \le -n$ $(1-\alpha) \le -1$ $\alpha \ge 2$

(u_n) $u_{n+1} - u_n$ 0

0 (u_n)

α v_1 $\frac{1}{\alpha}$ (v_n) 2

n $v_{n+1} = \frac{1}{\alpha}v_n$ $\frac{1}{\alpha}$ (v_n)

$$u_{n+1} = \frac{n+1}{\alpha n}u_n \qquad v_{n+1} = \frac{1}{\alpha(n+1)}u_{n+1} \qquad v_n = \frac{1}{\alpha n}u_n$$

$$v_{n+1} = \frac{1}{\alpha(n+1)}\frac{n+1}{\alpha n}u_n = \frac{1}{\alpha}(\frac{1}{\alpha n}u_n) = \frac{1}{\alpha}v_n$$

$$v_1 = \frac{1}{\alpha}u_1 = \frac{1}{\alpha^2} \qquad (v_n)$$

$\lim\limits_{x\to+\infty} u_n$ u_n v_n α n

$$v_n = \frac{1}{\alpha^2}(\frac{1}{\alpha})^{n-1} = (\frac{1}{\alpha})^{n-3} = \alpha^{3-n} \qquad v_n = v_1(q)^{n-1} \qquad (v_n)$$

$$\ln u_n = 4\ln\alpha + \ln n - n\ln\alpha = 4\ln\alpha + n(\frac{\ln n}{n} - \ln\alpha) \qquad u_n = \alpha n.v_n$$

$$\lim_{n\to+\infty} u_n = 0 \qquad \lim_{n\to+\infty} \ln u_n = -\infty$$

S_n α n 3

$u_n = \alpha n.v_n$ $S_n = u_1 + \frac{1}{2}u_2 + \frac{1}{3}u_3 + ... + \frac{1}{n}u_n$

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$$S_n = v_1\left[\frac{1-q^n}{1-q}\right] = \lim_{x\to+\infty}\frac{1-\left(\frac{1}{\alpha}\right)^n}{\alpha-1} \qquad S_n = \alpha(v_1+v_2+v_3+.....+v_n)$$

$\lim_{x\to+\infty} S_n = \frac{1}{2016}$ α

$\lim_{n\to+\infty}\left(\frac{1}{\alpha}\right)^n = 0$ $\alpha-1=2016$ $\lim_{n\to+\infty}\frac{1-\left(\frac{1}{\alpha}\right)^n}{\alpha-1} = \frac{1}{2016}$ $\lim_{n\to+\infty} S_n = \frac{1}{2016}$

$\alpha = 2017$

2 2016 48

$[1;+\infty[$ **f** **1**

$f(x) = \frac{x^2}{2x-1}$ $[1;+\infty[$ f

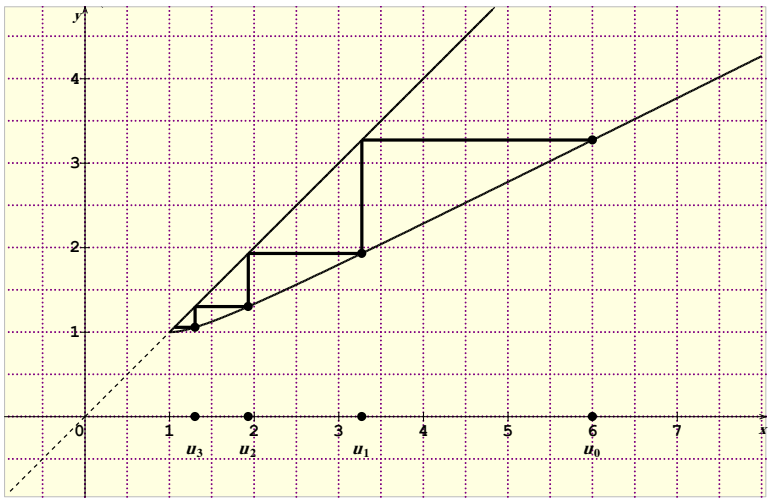
$f'(x) = \frac{2x(2x-1)-2(x^2)}{(2x-1)^2} = \frac{2x(x-1)}{(2x-1)^2}$

$2x(x-1)$ $f'(x)$

x	$-\infty$	0	1	$+\infty$
$2x(x-1)$		0	0	

$2x(x-1)\geq 0$ $[1;+\infty[$ f

(u_n) **2**



(u_n) **2**

1 (u_n)

$1\leq u_n\leq 6$ **n**

P(0) •

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$u_0 = 6$ $1 \le u_0 \le 6$ n=0

$1 \le u_{n+1} \le 6$ P(n+1) $1 \le u_n \le 6$ P(n)

$1 \le u_{n+1} \le \frac{36}{11}$ **f** $f(1) \le f(u_n)$ $f(6)$ $1 \le u_n \le 6$

$n \in$ $1 \le u_n \le 6$ $\frac{36}{11}$ 6 $1 \le u_{n+1} \le 6$

(u_n)

$u_{n+1} - u_n$ (u_n)

$$u_{n+1} - u_n = f(u_n) - u_n = \frac{u^2_n}{2u_n - 1} - u_n = \frac{-u_n^2 + u_n}{2u_n - 1} = \frac{u_n(-u_n + 1)}{2u_n - 1}$$

$u_n = 1$ $u_n = 0$ $u_{n+1} - u_n = 0$

u_n	$-\infty$	0	1	$+\infty$
		0	0	
	(u_n)	(u_n)		(u_n)

(u_n) $1 \le u_n \le 6$

(u_n)

1 (u_n)

2 (w_n) **3**

$n \in$ $w_{n+1} = 2.w_n$ 2 (w_n)

$w_n = \ln(v_n)$ $v_n = \frac{u_n - 1}{u_n}$

$$w_{n+1} = \ln(v_{n+1}) = \ln\left(\frac{u_{n+1} - 1}{u_{n+1}}\right) = \ln\left(\frac{\frac{u^2_n}{2u_n - 1} - 1}{\frac{u^2_n}{2u_n - 1}}\right) = \ln\left(\frac{u_n - 1}{u_n}\right)^2 = 2\ln\left(\frac{u_n - 1}{u_n}\right) = 2w_n$$

$$w_0 = \ln(v_0) = \ln\left(\frac{u_0 - 1}{u_0}\right) = \ln\frac{5}{6}$$

n v_n w_n

$w_n = w_0.q^n = \ln\frac{5}{6}.(2)^n$ $n \in$ (w_n)

$$v_n = e^{2^n.\ln\frac{5}{6}} = \left(\frac{5}{6}\right)^{2^n} \qquad v_n = e^{w_n} \qquad w_n = \ln(v_n)$$

$\lim\limits_{x\to+\infty} u_n$ $\qquad u_n = \dfrac{1}{1-\left(\frac{5}{6}\right)^{2^n}}$

$$u_n = \frac{1}{1-\left(\frac{5}{6}\right)^{2^n}} \qquad u_n = \frac{1}{1-v_n} \qquad v_n - 1 = -\frac{1}{u_n} \qquad v_n = \left(\frac{5}{6}\right)^{2^n} \qquad v_n = \frac{u_n - 1}{u_n}$$

$$\lim_{x\to+\infty}\left(\frac{5}{6}\right)^{2^n} = 0 \qquad \lim_{x\to+\infty} u_n = \lim_{x\to+\infty} \frac{1}{1-\left(\frac{5}{6}\right)^{2^n}} = 1$$

4 S_n n

$$S_n = \frac{1}{w_0} + \frac{1}{w_1} + \ldots + \frac{1}{w_n} = \frac{1}{w_0} + \frac{1}{w_0.q} + \ldots.. + \frac{1}{w_0.q^n} = \frac{1}{w_0}\left[1 + \frac{1}{q} + \frac{1}{q^2} + \ldots.. + \frac{1}{q^n}\right]$$

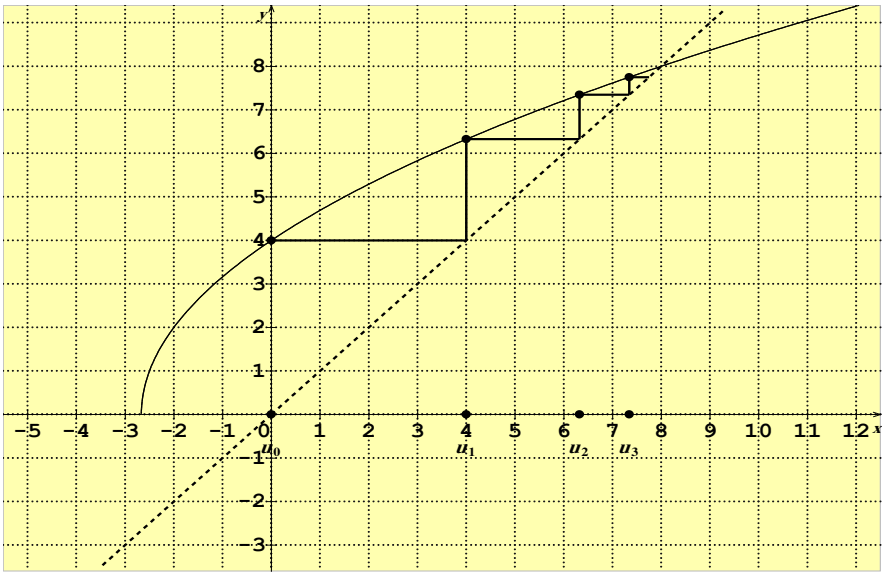
$\left[1 + \frac{1}{q} + \frac{1}{q^2} + \ldots.. + \frac{1}{q^n}\right]$ 1 $\frac{1}{q}$

$$S_n = \frac{1}{w_0}\left[\frac{1-\left(\frac{1}{q}\right)^{n+1}}{1-\frac{1}{q}}\right] = \frac{1}{\ln\frac{5}{6}}\left[\frac{1-\left(\frac{1}{2}\right)^{n+1}}{1-\frac{1}{2}}\right] = \frac{2}{\ln\frac{5}{6}}\left(1-\left(\frac{1}{2}\right)^{n+1}\right)$$

2 2015 49

1

u_3 u_2 u_1 u_0



(u_n)

8 (u_n) 1

$0 \quad u_n \quad 8$ n **2**

$u_0 = 0$ $0 \quad u_0 \quad 8$ $n = 0$

$0 \quad u_n \quad 8$ n

$0 \quad u_{n+1} \quad 8$ n

$0 \quad u_n \quad 8$ n

h $h(0)$ $h(u_n)$ $h(8)$

$[0;8[$ $[4;8[$ $0 \quad u_{n+1} \quad 8 \quad 4 \quad u_{n+1} \quad 8$

$$u_{n+1} - u_n = \frac{(8 - u_n)(u_n + 2)}{\sqrt{6u_n + 16} + u_n}$$ n

$$u_{n+1} - u_n = \sqrt{6u_n \quad 16} \quad u_n \quad \frac{(\sqrt{6u_n \quad 16} \quad u_n)(\sqrt{6u_n \quad 16} \quad u_n)}{(\sqrt{6u_n \quad 16} \quad u_n)}$$

$$u_{n+1} - u_n \quad \frac{(\sqrt{6u_n \quad 16})^2 \quad u_n^2}{(\sqrt{6u_n \quad 16} \quad u_n)} \quad \frac{u_n^2 \quad 6u_n \quad 16}{(\sqrt{6u_n \quad 16} \quad u_n)}$$

$$u_{n+1} - u_n = \frac{(8 - u_n)(u_n + 2)}{\sqrt{6u_n + 16} + u_n}$$ $(8 - u_n)(u_n + 2)$ $u_n^2 \quad 6u_n \quad 16$

(u_n)

$u_{n+1} - u_n$ (u_n)

$$u_{n+1} - u_n = \frac{(8 - u_n)(u_n + 2)}{\sqrt{6u_n + 16} + u_n}$$

$(8 - u_n)(u_n + 2)$

u_n	$-\infty$	2	8	$+\infty$
$(8-u_n)(u_n+2)$		0	0	

$0 \quad u_n \quad 8$

(u_n)

$0 \quad 8 - u_{n+1} \quad \frac{1}{2}(8 - u_n)$ n **3**

(1) $0 \quad 8 \quad u_{n+1}$ $0 \quad 8 \quad u_{n+1} \quad 8 \quad -8 \quad u_{n+1} \quad 0$ $0 \quad u_{n+1} \quad 8$

(2) $8 - u_{n+1} \quad \frac{1}{2}(8 - u_n)$

$$8-u_{n+1} \quad \frac{1}{2}(8-u_n) \quad 0 \qquad (2)$$

$$8-u_{n+1} \quad \frac{1}{2}(8-u_n) \quad 8 \quad u_n \quad 2\sqrt{6u_n \quad 16} \quad \frac{(8 \quad u_n)^2 \quad 4(\sqrt{6u_n \quad 16})^2}{(8 \quad u_n \quad 2\sqrt{6u_n \quad 16})}$$

$$\frac{(8 \quad u_n)^2 \quad 4(\sqrt{6u_n \quad 16})^2}{(8 \quad u_n \quad 2\sqrt{6u_n \quad 16})} \quad \frac{u_n^2 \quad 8u_n}{(8 \quad u_n \quad 2\sqrt{6u_n \quad 16})} \quad \frac{u_n(u_n \quad 8)}{(8 \quad u_n \quad 2\sqrt{6u_n \quad 16})}$$

$$0 \quad u_n \quad 8$$

n $\qquad 0 \quad 8-u_{n+1} \quad \frac{1}{2}(8-u_n) \qquad (2) \quad (1)$

$.\lim_{x\to+\infty} u_n \qquad 0 \quad 8-u_n \quad 8\left(\frac{1}{2}\right)^n$ n

n $\qquad 0 \quad 8-u_{n+1} \quad \frac{1}{2}(8-u_n)$

$$0 \quad 8-u_1 \quad \frac{1}{2}(8-u_0) \qquad n=0$$

$$0 \quad 8-u_2 \quad \frac{1}{2}(8-u_1) \qquad n=1$$

$$0 \quad 8-u_n \quad \frac{1}{2}(8-u_{n\ 1}) \qquad n-1$$

$0 \quad 8-u_n \quad 8\left(\frac{1}{2}\right)^n \qquad 0 \quad 8-u_1 \quad \left(\frac{1}{2}\right)^n (8-u_0)$

$\lim_x 8\left(\frac{1}{2}\right)^n \quad 0 \qquad 0 \quad 8-u_n \quad 8\left(\frac{1}{2}\right)^n$

$\lim_x u_n \quad 8 \qquad \lim_x (8-u_n) \quad 0$

1 2014 50

5^n **16** 1

$5^4 \equiv 1[16] \quad 5^3 \equiv 13[16] \quad 5^2 \equiv 9[16] \quad 5^1 \equiv 5[16] \quad 5^0 \equiv 1[16]$

4 16 5^n

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$n=$	$4k$	$4k+1$	$4k+2$	$4k+3$
$5^n \equiv$	1	5	9	13

$C_n = D_P$ **n** k $p=4k+2$ **2**

n $5^{4k+2} \equiv 9[16]$ $p=4k+2$

$D_P = 5^p$ $C_n = 16n+9$ $C_n = D_P$ $5^{4k+2} \equiv 16n+9$

$p=6$ **n**

$n=976$ $16n+9=15625$ $5^6 \equiv 16n+9$ $p=6$

f(x) **f** **3**

$f(0)=16$ $\lim\limits_{x\ +} f(x) = \lim\limits_{x\ +} 5^{4x}$

f $f'(x) = 4\ln 5.5^{4x+2}$ 0

x	0	$+\infty$
$f'(x)$	+	
$f(x)$	16 ↗	$+\infty$

$f(x)$ 0 $f(x)$ 16

$u_n = \dfrac{5^{(4n+2)}-9}{16}$ $n \in$ **4**

$u_0 = 1$ $u_0 = \dfrac{5^{(2)}-9}{16} = 1$ $n=0$

$u_n = \dfrac{5^{(4n+2)}-9}{16}$ n

$u_{n+1} = \dfrac{5^{(4n+6)}-9}{16}$ n

$$u_{n+1} = 5^4\left[\frac{5^{(4n+2)}-9}{16}+\frac{9}{16}\right]-\frac{9}{16} = \frac{5^{(4n+2)}.5^4-9.5^4}{16}+\frac{9.5^4}{16}-\frac{9}{16} = \frac{5^{(4n+6)}-9}{16}$$ n

n $u_n = \dfrac{5^{(4n+2)}-9}{16}$

u_n **n**

$5^{4k+2}-9 \equiv 0[16]$ $5^{4k+2} \equiv 9[16]$ 2

u_n $u_n = \dfrac{5^{4k+2}-9}{16} \equiv 0[16]$

(u_n) **5**

f (u_n) $u_n = \dfrac{1}{16} f(n)$

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2 2014 51

$f(x)-x$ x **1 I**

$-\ln(x-1)$ $f(x)-x=-\ln(x-1)$

1 x 2 $-\ln(x-1)$ 0 x 2 $-\ln(x-1)$ 0 $x=2$ $-\ln(x-1)=0$

f **2**

$$f'(x)=1-\frac{1}{(x-1)}=\frac{x-2}{x-1} \qquad f(x)=x-\ln(x-1)$$

1 x 2 $f'(x)$ 0 x 2 $f'(x)$ 0 $x=2$ $f'(x)=0$

. $]1;2]$ $[2;+\infty[$ f

$f(x)\in[2:e+1]$ $x\in[2:e+1]$

$[0;+\infty[$ f $f(x)\in[f(2);f(e+1)]$ $x\in[2:e+1]$

$f(e+1)=e+1-1=e$ $f(2)=2-\ln 3$

$[2:e+1]$ $[f(2);f(e+1)]$ $f(x)\in[2:e+1]$

$u_n\in[2:e+1]$ **n** **1 II**

$u_0=e+1$ $u_0\in[2:e+1]$ n=0

$u_{n+1}\in[2:e+1]$ $u_n\in[2:e+1]$

$u_{n+1}=f(u_n)$ $u_{n+1}=u_n-\ln(u_n-1)$

$[0;+\infty[$ f $f(u_n)\in[f(2);f(e+1)]$ $u_n\in[2:e+1]$

$f(e+1)=e+1-1=e$ $f(2)=2-\ln 3$

$[2:e+1]$ $[f(2);f(e+1)]$ $f(u_n)\in[2:e+1]$

n $u_n\in[2:e+1]$

(u_n) **2**

$u_n\in[2:e+1]$ $u_{n+1}-u_n=-\ln(u_n-1)$ 0 $u_{n+1}=u_n-\ln(u_n-1)$

(u_n)

(u_n) **3**

2 (u_n)

$=f(\ )$ $u_{n+1}=f(u_n)$ $\lim\limits_{x} u_n$

2 $\ln(\ -1)=0$ $=f(\ )$

1 2013 52

$\frac{1}{2}$ (v_n) **1**

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n $v_{n+1}=\frac{1}{2}.v_n$ $\frac{1}{2}$ (v_n)

$$v_{n+1}=\frac{1}{2}\ln u_{n+1}+\frac{1}{2}=\frac{1}{2}(\ln\sqrt{\frac{u_n}{e}}+1)=\frac{1}{2}(\frac{1}{2}\ln\frac{u_n}{e}+1)=\frac{1}{2}(\frac{1}{2}\ln u_n-\frac{1}{2}+1)=\frac{1}{2}v_n \qquad v_n=\frac{1}{2}\ln u_n+\frac{1}{2}$$

$$v_0=\frac{1}{2}\ln u_0+\frac{1}{2}=\frac{1}{2}\ln e^2+\frac{1}{2}=\frac{3}{2}$$

n u_n v_n **2**

$v_n=\frac{3}{2}.\left(\frac{1}{2}\right)^n$ $v_n=v_0.q^n$ q (v_n)

$u_n=e^{3\left(\frac{1}{2}\right)^n-1}$ $u_n=e^{2v_n-1}$ $\ln u_n=2v_n-1$ $v_n=\frac{1}{2}\ln u_n+\frac{1}{2}$

$\lim\limits_{x\to+\infty} S_n$ S_n n **3**

$$S_n=v_0+v_1+...+v_n=v_0\left[\frac{1-q^{n+1}}{1-q}\right]=\frac{3}{2}\left[\frac{1-\left(\frac{1}{2}\right)^{n+1}}{1-\frac{1}{2}}\right]=3\left[1-\left(\frac{1}{2}\right)^{n+1}\right]$$

$$\lim_{x\to+\infty}\left(\frac{1}{2}\right)^{n+1}=0 \qquad \lim_{x\to+\infty} S_n=\lim_{x\to+\infty}3\left[1-\left(\frac{1}{2}\right)^{n+1}\right]=3$$

$\lim\limits_{x\to+\infty} P_n$ $P_n=u_0\times u_1\times..\times u_n$ **n** **4**

$$P_n=u_0\times u_1\times..\times u_n=e^{2v_0-1}\times e^{2v_1-1}\times..\times e^{2v_n-1}=e^{2(v_0+v_1+...+v_n)-(n+1)}=e^{2(S_n)-(n+1)}$$

$$\lim_{x\to+\infty}\left(e^{-(n+1)}\right)=0 \qquad \lim_{x\to+\infty} P_n=\lim_{x\to+\infty} e^{2(S_n)-(n+1)}=0$$

1 2011 53

u_n 1 $u_n=1+\frac{1}{n(n+2)}$ $n\in$ * **1**

$$u_n=\frac{(n+1)^2}{n(n+2)}=\frac{n(n+2)+1}{n(n+2)}=1+\frac{1}{n(n+2)}$$

u_n 1 $\frac{1}{n(n+2)}+1$ 1 $\frac{1}{n(n+2)}$ 0 $n\in$ *

(u_n) **2**

$u_{n+1}-u_n=1+\frac{1}{(n+1)(n+3)}-1-\frac{1}{n(n+2)}=\frac{-2n-3}{n(n+2)(n+1)(n+3)}$ 0 (u_n)

1

$$\lim_{x\to+\infty} u_n = 1 + \lim_{x\to+\infty} \frac{1}{n(n+2)} = 1+0 = 1$$

$P_n = \frac{2n+2}{n+2}$ $n \in$ * **3**

$P_1 = \frac{(1+1)^2}{1+2} = \frac{4}{3}$ $P_1 = \frac{2(1)+2}{1+2} = \frac{4}{3}$ n=1

$P_{n+1} = \frac{2n+4}{n+3}$ $P_n = \frac{2n+2}{n+2}$

$P_n = u_1 \times u_2 \times u_3 \times \ldots \times u_n$

$$P_{n+1} = u_1 \times u_2 \times u_3 \times \ldots \times u_{n+1} = P_n \times u_{n+1} = \left(\frac{2n+2}{n+2}\right).\frac{(n+2)^2}{(n+1)(n+3)} = \frac{2(n+1)(n+2)^2}{(n+1)(n+2)(n+3)} = \frac{2n+4}{n+3}$$

$P_n = \frac{2n+2}{n+2}$ $n \in$ * $P_n = u_1 \times u_2 \times u_3 \times \ldots \times u_n$

S_n P_n **4**

$S_n = v_1 + v_2 + v_3 + \ldots + v_n$ $v_n = \ln(u_n)$

$S_n = \ln u_0 + \ln u_1 + \ldots + \ln u_n = \ln(u_0.u_1.\ldots.u_n) = \ln(P_n)$

$+\infty$ **n** S_n

$\lim_{x\to+\infty}(P_n) = 2$ $\lim_{x\to+\infty} S_n = \lim_{x\to+\infty} \ln(P_n) = \ln 2$

1 2008 54

f **I**

$f(x) = \frac{x^2+5}{x+2}$ 2 $+\infty$ f

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \frac{x^2}{x} = \lim_{x\to+\infty} x = +\infty \qquad \lim_{x\longrightarrow -2} (x+2) = 0^+ \qquad \lim_{x\longrightarrow -2} f(x) = \lim_{x\longrightarrow -2} \frac{9}{x+2} = +\infty$$

f

$$f'(x) = \frac{2x(x+2)-1(x^2+5)}{(x+2)^2} = \frac{x^2+4x-5}{(x+2)^2} = \frac{(x-1)(x+5)}{(x+2)^2} \qquad f(x) = \frac{x^2+5}{x+2}$$

$x = -5$ $x = 1$ $(x-1)(x+5) = 0$ $f'(x) = 0$

-2 x 1 $(x-1)(x+5)$ 0 $f'(x)$ 0

$x \in]1;+\infty[$ $(x-1)(x+5)$ 0 $f'(x)$ 0

f

x	2	1	$+\infty$

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f'(x)	0
f(x)	$+\infty$ ↘ 2 ↗ $+\infty$

C_f $y=x-2$ D

$\lim\limits_{x\to+\infty}[f(x)-y]=0$ C_f $y=x-2$ D

$$\lim_{x\to+\infty}[f(x)-y]=\lim_{x\to+\infty}\left[\frac{x^2+5}{x+2}-(x-2)\right]=\lim_{x\to+\infty}\frac{9}{x+2}=0$$

II D C_f

$\left[1;\frac{5}{2}\right]$ $\left[1;\frac{5}{2}\right]$

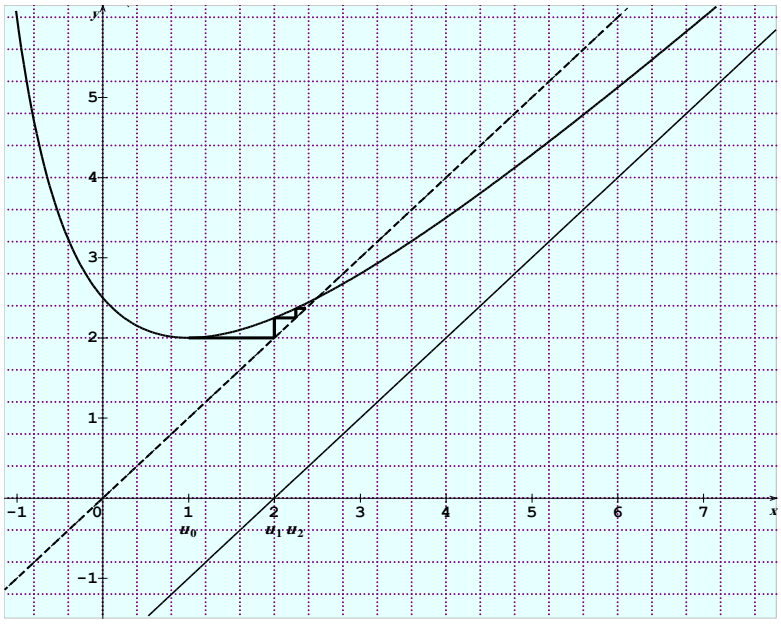
1 $f(x)$ $\frac{5}{2}$ 1 x $\frac{5}{2}$

$\left[1;\frac{5}{2}\right]$ f $f(1)$ $f(x)$ $f(\frac{5}{2})$ 1 x $\frac{5}{2}$

1 $f(x)$ $\frac{5}{2}$ 2 $f(x)$ $\frac{29}{18}$ $f(\frac{5}{2})=\frac{29}{18}$ $f(1)=2$

$\left[1;\frac{5}{2}\right]$ $\left[2;\frac{29}{18}\right]$

C_f U_2 U_1 U_0 II



U_n

$\frac{5}{2}$

n U_{n+1} $f(U_n)$ $U_0=1$ U_n

$1 \le U_n \le \frac{5}{2}$ **n**

$u_0 = 1$ $1 \le U_0 \le \frac{5}{2}$ $n = 0$

$1 \le U_n \le \frac{5}{2}$ n

$1 \le U_{n+1} \le \frac{5}{2}$ n

$1 \le U_n \le \frac{5}{2}$ n

$\left[1;\frac{5}{2}\right]$ f $f(1) \le f(U_n) \le f(\frac{5}{2})$

$\left[1;\frac{5}{2}\right]$ $\left[2;\frac{29}{18}\right]$ 1 U_{n+1} $\frac{5}{2}$ 2 $f(U_n)$ $\frac{29}{18}$

U_n

U_0 U_1 $\left[1;\frac{5}{2}\right]$ U_n

$\lim\limits_{n\to+\infty} U_n$ **U_n**

$\frac{5}{2}$ U_n

$f(\alpha) = \alpha$ $\lim\limits_{x\to+\infty} U_n = \alpha$

$\lim\limits_{x\to+\infty} U_n = \frac{5}{2}$ $\alpha = \frac{5}{2}$ $2\alpha = 5$ $\frac{\alpha^2+5}{\alpha+2} = \alpha$ $f(\alpha) = \alpha$

2 2008 55

$[0;2]$ **f** **1**

$f'(x) = \frac{2(x+2)-(2x+3)}{(x+2)^2} = \frac{1}{(x+2)^2}$ $f(x) = \frac{2x+3}{x+2}$

$[0;2]$ f $f'(x)$ 0

f

x	0	2
f'(x)		
f(x)		$\frac{7}{4}$

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	$\frac{3}{2}$

2

$u_2 \quad u_1 \quad u_0$ (C)

$f(x) \in [0;2] \quad x \in [0;2]$

$\left[\frac{3}{2};\frac{7}{4}\right]$ $[0;2]$ f

$[0;2]$ $\left[\frac{3}{2};\frac{7}{4}\right]$ $f(x) \in [0;2]$

$u_2 \quad u_1$ (u_n) 2

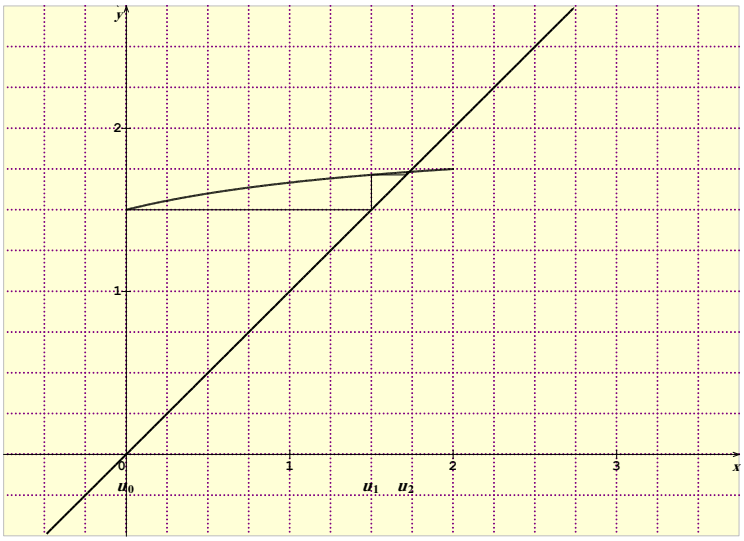
$u_{n+1} = f(u_n)$ $u_0 = 0$ (u_n)

1 $[0;2]$

$$\begin{cases} u_1 = \frac{3}{2} \\ u_2 = f(u_1) = f(\frac{3}{2}) = \frac{12}{7} \end{cases} \qquad \begin{cases} u_0 = 0 \\ u_1 = f(u_0) = f(0) = \frac{3}{2} \end{cases} \qquad \begin{cases} u_0 = 0 \\ u_{n+1} = f(u_n) \end{cases}$$

$$\begin{cases} u_0 = 0 \\ u_{n+1} = f(u_n) \end{cases} \quad (u_n)$$

$u_2 \quad u_1 \quad u_0$ (C)



(u_n)

$u_2 \quad u_1 \quad u_0$

$y = x$ (D) (C)

$0 \le u_n \le \sqrt{3}$ **n** **3**

$u_0 = 0$ $0 \le u_0 \le \sqrt{3}$ $n = 0$

$0 \le u_n \le \sqrt{3}$ n

$0 \le u_{n+1} \le \sqrt{3}$ n

$0 \le u_n \le \sqrt{3}$ n

$[0;2]$ f $f(0) \le f(u_n) \le f(\sqrt{3})$

$\left[0;\sqrt{3}\right]$ $\left[\frac{3}{2};\sqrt{3}\right]$ $0 \le u_{n+1} \le \sqrt{3}$ $\frac{3}{2} \le f(u_n) \le \sqrt{3}$

(u_n) u_{n+1} u_n **n**

$u_{n+1} - u_n$ 0 u_{n+1} u_n n

$$u_{n+1} - u_n = f(u_n) - u_n = \frac{2u_n + 3}{u_n + 2} - u_n = \frac{-u_n^2 + 3}{u_n + 2} = \frac{-(u_n + \sqrt{3})(u_n - \sqrt{3})}{u_n + 2}$$

$0 \le u_n \le \sqrt{3}$ $u_{n+1} - u_n$ 0

$n \in$ * $(u_{n+1} - \sqrt{3}) \le \frac{2-\sqrt{3}}{u_n + 2}(u_n - \sqrt{3})$

$$(u_{n+1} - \sqrt{3}) = \frac{2u_n + 3}{u_n + 2} - \sqrt{3} = \frac{(2-\sqrt{3})}{u_n + 2}(u_n - \sqrt{3})$$

$(u_{n+1} - \sqrt{3}) \le \frac{2-\sqrt{3}}{u_n + 2}(u_n - \sqrt{3})$ $\frac{2-\sqrt{3}}{2+\sqrt{3}}$ $\frac{2-\sqrt{3}}{u_n + 2}$ $\frac{2-\sqrt{3}}{2}$ 0 $\frac{2-\sqrt{3}}{u_n + 2}$ 1

$\left|(u_{n+1} - \sqrt{3})\right| \le k\left|(u_n - \sqrt{3})\right|$ $]0;1[$ k

$k \le \left|\frac{2-\sqrt{3}}{u_n + 2}\right|$ $\left|(u_{n+1} - \sqrt{3})\right| \le k\left|(u_n - \sqrt{3})\right|$ $(u_{n+1} - \sqrt{3}) \le \frac{2-\sqrt{3}}{u_n + 2}(u_n - \sqrt{3})$

$\lim\limits_{x \to +\infty} u_n$ $\left|(u_n - \sqrt{3})\right| \le k^n \left|(u_0 - \sqrt{3})\right|$ $n \in$ *

$\left|(u_{n+1} - \sqrt{3})\right| \le k\left|(u_n - \sqrt{3})\right|$

$\left|(u_2 - \sqrt{3})\right| \le k\left|(u_1 - \sqrt{3})\right|$ n=1 $\left|(u_1 - \sqrt{3})\right| \le k\left|(u_0 - \sqrt{3})\right|$ n=0

$\left|(u_n - \sqrt{3})\right| \le k\left|(u_{n-1} - \sqrt{3})\right|$ n-1 $\left|(u_3 - \sqrt{3})\right| \le k\left|(u_2 - \sqrt{3})\right|$ n=2

$\left|(u_n - \sqrt{3})\right| \le k^n \left|(u_0 - \sqrt{3})\right|$

$\left|(u_n - \sqrt{3})\right| \le k^n \left|(u_0 - \sqrt{3})\right|$

$\lim\limits_{x \to +\infty} u_n = \sqrt{3}$ $\lim\limits_{x \to +\infty} \left|(u_n - \sqrt{3})\right| = 0$ $\lim\limits_{x \to +\infty} k^n = 0$ $]0;1[$ k

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$505x-673y=1.....(E)$ **1**

$2020=4\times505$ $2019=3\times673$ $(505.4-673.3=1.....(E')$

$(4;3)$ (E)

$$\begin{cases}505x-673y=1.....(E)\\505.4-673.3=1.....(E')\end{cases}$$

$505(x-4)-673(y-3)=0$

$505(x-4)-673(y-3)=0$ $505(x-4)=673(y-3).....(*)$

$(*)$ $673/505(x-4)$ $673/(x-4)$ 367 505

$x-4=673k$ $x=673k+4$ k

x $(*)$ $505(673k)=673(y-3)$ $y=505k+3$

(E) $(x;y)=(673k+4;505k+3)$ k

2 $(x;y)$ (E) x y

y x $(x.y)=(673k+4)(505k+3)=339865k^2+1178k+12>0$

$\Delta<0$ k^2

3 u_α α v_β β

(u_n) (v_n)

$$\begin{cases}u_0=3\\u_{n+1}=u_n+505\end{cases}$$ $$\begin{cases}v_0=4\\v_{n+1}=v_n+673\end{cases}$$

$$(1)...\begin{cases}u_0=3\\u_{n+1}-u_n=505\end{cases}$$ $$(2)....\begin{cases}v_0=4\\v_{n+1}-v_n=673\end{cases}$$

(1) (2)

(u_n) 505 3 $u_\alpha=u_0+\alpha r$ $u_\alpha=3+505\alpha$

(v_n) 673 4 $v_\beta=v_0+\beta r'$ $v_\beta=4+673\beta$

4 (w_n)

(u_n) (v_n) (w_n) $u_\alpha=v_\beta$

$u_\alpha=v_\beta$ $3+505\alpha=4+673\beta$ $505\beta-673\alpha=1$ $\alpha=y$ $\beta=x$

1 $(\beta;\alpha)=(673k+4;504k+3)$ k

(w_n) $w_k=3+505(673k+4)=339865k+2023$

$w_{k+1}-w_k=339865(k+1)-339865k=339865$

(w_n) 339865 2023

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P n

$$X_n = \frac{1}{505}(w_n - 2023) \qquad P = X_1.X_2.X_3.....X_n$$

$$X_n = \frac{1}{505}(w_n - 2023) = \frac{1}{505}(339192n) = \frac{1}{505}(505 \times 673n) = 673n$$

$$P = X_1.X_2.X_3.....X_n = (673)^n(1.2.3.....n) = (673)^n.n!$$

2 2019 57

$\sqrt{u_{n+1}} - \sqrt{u_n} = 1$ n **1**

$u_1 = 0$ (u_n)

$u_{n+1} = u_n + 2\sqrt{u_n} + 1$ n

$\left(\sqrt{u_{n+1}}\right)^2 = \left(\sqrt{u_n} + 1\right)^2$ $\left(\sqrt{u_{n+1}}\right)^2 = \left(\sqrt{u_n}\right)^2 + 2\sqrt{u_n} + 1$ $u_{n+1} = u_n + 2\sqrt{u_n} + 1$

$\sqrt{u_{n+1}} - \sqrt{u_n} = 1$ $\left(\sqrt{u_{n+1}}\right) = \left(\sqrt{u_n} + 1\right)$

n u_n

$k_{n+1} - k_n = 1$ $\sqrt{u_{n+1}} - \sqrt{u_n} = 1$ $\sqrt{u_{n+1}} = k_{n+1}$ $\sqrt{u_n} = k_n$

$k_n = n - 1$ $k_1 = 0$ 1 (k_n)

$u_n = (n-1)^2$ $\sqrt{u_n} = k_n = n - 1$

$u_n = n(n-2) + 1$ n **2**

$u_n = n^2 - 2n + 1 = (n-1)^2$ $u_n = n(n-2) + 1$

$(n-5)$ $(n-2)$ n **3**

$\frac{n-5}{n-2}$ $(n-5)$ $(n-2)$

3 $(n-2)$ $\frac{3}{n-2}$ $\frac{n-5}{n-2} = 1 - \frac{3}{n-2}$

$n \in \{1;3;5\}$ $(n-2) \in D_3 = \{-3;-1;1;3\}$ 3 $(n-2)$

$PGCD(n-2;u_n) = 1$ **4**

$u_n - n(n-2) = 1$ $u_n = n(n-2) + 1$

$u_n - n(n-2) = 1$ $(1;n)$

$PGCD(n-2;u_n) = 1$

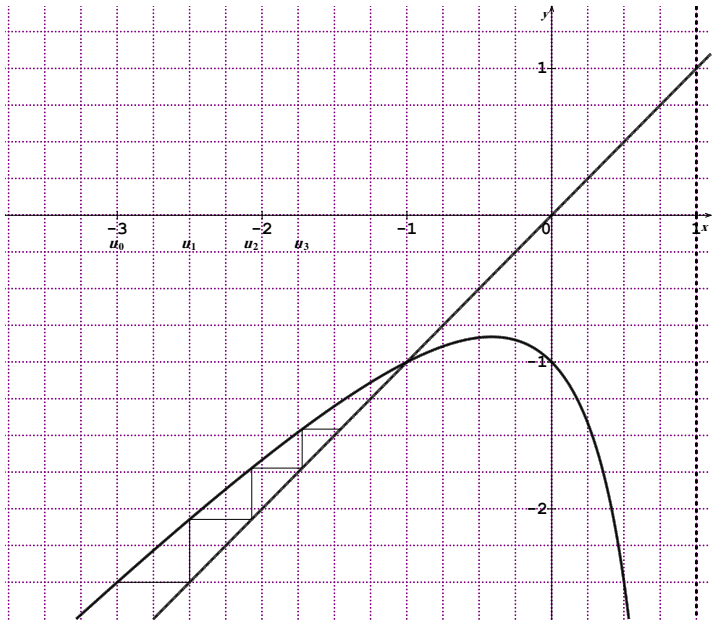
$(n-5)u_n$ $(n-2)(n^2+1)$ n

$PGCD(n-2;u_n) = 1$ $(n-5)u_n$ $(n-2)(n^2+1)$

$n \geq 2$ $n \in \{3;5\}$ 3 $n \in \{1;3;5\}$ $(n-5)$ $(n-2)$

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u_3 u_2 u_1 u_0 **1**



(u_n)

1 (u_n)

$-3 \le u_n \le -1$ **n** **2**

P(0)

$u_0 = -3$ $-3 \le u_0 \le -1$ n=0

$-3 \le u_{n+1} \le -1$ P(n+1) $-3 \le u_n \le -1$ P(n)

f $f(-3) \le f(u_n)$ $f(-1)$ $-3 \le u_n \le -1$

$f(-1) = -1$ $f(-3) = 5$ $5 \le u_{n+1} \le -1$

$-3 < 5$ $-3 \le u_{n+1} \le -1$ $5 \le u_{n+1} \le -1$

$n \in$ $-3 \le u_n \le -1$

$u_{n+1} + 1 \ge \frac{3}{4}(u_n + 1)$ **n** **3**

$(u_{n+1} + 1) - \frac{3}{4}(u_n + 1) \ge 0$ $u_{n+1} + 1 \ge \frac{3}{4}(u_n + 1)$ n

$$(u_{n+1} + 1) - \frac{3}{4}(u_n + 1) = \frac{u_n^2 + 1}{u_n - 1} - \frac{3}{4}(u_n + 1) = \frac{(u_n + 3)(u_n + 1)}{4(u_n - 1)} \ge 0$$

$-3 \le u_n \le -1$ $(u_n - 1) < 0$ $(u_n + 1) < 0$ $(u_n + 3) > 0$

$\lim\limits_{x \to +\infty} u_n$ $u_n + 1 \ge -2\left(\frac{3}{4}\right)^n$ **n**

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$$u_{n+1}+1\geq\frac{3}{4}(u_n+1)$$

$$u_2+1\geq\frac{3}{4}(u_1+1)\quad n=1 \qquad u_1+1\geq\frac{3}{4}(u_0+1)\quad n=0$$

$$u_n+1\geq\frac{3}{4}(u_{n-1}+1)\quad n\quad 1$$

$$u_n+1\geq-2\left(\frac{3}{4}\right)^n \qquad u_n+1\geq\left(\frac{3}{4}\right)^n(u_0+1)$$

$\lim\limits_{x\to+\infty} u_n$

$$-2\left(\frac{3}{4}\right)^n\leq u_n+1\leq 0 \qquad -2\leq u_n+1\leq 0 \qquad u_n+1\geq-2\left(\frac{3}{4}\right)^n$$

$$\lim_{x\to+\infty} u_n=-1 \qquad \lim_{x\to+\infty}(u_n+1)=0 \qquad \left(\frac{3}{4}\right)<1 \qquad \lim_{x\to+\infty}\left(\frac{3}{4}\right)^n=0$$

$$8\left[\left(\frac{3}{4}\right)^{n+1}-1\right]\leq(u_0+1)+(u_1+1)+.....+(u_n+1)<0 \quad \mathbf{n} \qquad \mathbf{4}$$

$$u_n+1\geq-2\left(\frac{3}{4}\right)^n$$

$$u_1+1\geq-2\left(\frac{3}{4}\right)^1\quad n=1 \qquad u_0+1\geq-2\left(\frac{3}{4}\right)^0\quad n=0$$

$$u_n+1\geq-2\left(\frac{3}{4}\right)^n\quad n\quad 1$$

$$u_n+1\geq\left(\frac{3}{4}\right)^n(u_0+1)$$

$$(u_0+1)+(u_1+1)+.....+(u_n+1)\geq-2\left[\left(\frac{3}{4}\right)^0+\left(\frac{3}{4}\right)^1+.....+\left(\frac{3}{4}\right)^n\right]$$

$$T_n=\left(\frac{3}{4}\right)^0+\left(\frac{3}{4}\right)^1+.....+\left(\frac{3}{4}\right)^n$$

$$T_n=\frac{1-\left(\frac{3}{4}\right)^{n+1}}{1-\left(\frac{3}{4}\right)}=4\left[1-\left(\frac{3}{4}\right)^{n+1}\right]$$

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$$(u_0+1)+(u_1+1)+.....+(u_n+1)\geq 8\left[\left(\frac{3}{4}\right)^{n+1}-1\right]...(1)$$

$T_n>0$ $(u_0+1)+(u_1+1)+.....+(u_n+1)<0...(2)$

$$8\left[\left(\frac{3}{4}\right)^{n+1}-1\right]\leq (u_0+1)+(u_1+1)+.....+(u_n+1)<0$$ 2 1

$\lim\limits_{x\to+\infty} S_n$

$S_n=u_0+u_1+u_2+...+u_n<-n-1$ $(u_0+1)+(u_1+1)+.....+(u_n+1)<0$

$\lim\limits_{x\to+\infty} S_n=-\infty$ $\lim\limits_{x\to+\infty}(-n-1)=-\infty$

2 2017 59

$3u_n=7^{n+1}-4$ n **1**

$u_{n+1}=7u_n+8:n$ $u_0=1$ (u_n)

P(0)

$u_0=1$ $3u_0=3$ n=0

$3u_{n+1}=7^{n+2}-4$ P(n+1) $3u_n=7^{n+1}-4$ P(n)

$3u_{n+1}=7(3u_n)+24$ $u_{n+1}=7u_n+8:n$

$3u_{n+1}=7(7^{n+1}-4)+24$ $3u_n=7^{n+1}-4$

$3u_{n+1}=7^{n+2}-4$ $3u_{n+1}=7.7^{n+1}-28+24$

n $3u_n=7^{n+1}-4$

S_n n 2

7 1 $S_n=1+7+7^2.....+7^n$

$$S_n=1\left(\frac{7^{n+1}-1}{7-1}\right)=\frac{7^{n+1}-1}{6}$$

S'_n S_n

$3u_n=7^{n+1}-4$ $S_n'=u_0+u_1+.....+u_n$

$3S_n'=3u_0+3u_1+.....+3u_n$

$$3S_n'=\left[(7-4)+(7^2-4)+.....+(7^{n+1}-4)\right]=7(1+7+7^2+....7^n)-4(n+1)$$

$3S_n'=7S_n-4(n+1)$

$18\times S'_n=7^{n+2}-24n-31$ n

6 $3S_n'=7S_n-4(n+1)$

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$S_n = \frac{7^{n+1}-1}{6}$ $18S_n' = 7(6S_n) - 24(n+1)$

$18 \times S'_n = 7^{n+2} - 24n - 31$ $18S_n' = 7(6\frac{7^{n+1}-1}{6}) - 24(n+1)$

5 7^n n 3

$7^4 \equiv 1[5]$ $7^3 \equiv 3[5]$ $7^2 \equiv 4[5]$ $7^1 \equiv 2[5]$ $7^0 \equiv 1[5]$

4 5 7^n

$n =$	$4k$	$4k+1$	$4k+2$	$4k+3$
$7^n \equiv$	1	2	4	3

5 S'_n n

18 $18S'_n \equiv 0[5]$ $S'_n \equiv 0[5]$ 5 S'_n

$7^{n+2} + n - 1 \equiv 0[5]$ $7^{n+2} - 24n - 31 \equiv 0[5]$

$(k' \in \quad) n = 20k' + 12$ $k \equiv 3[5]$ $7^{n+2} + n - 1 \equiv 0[5]$ $n = 4k$ 1

$(k' \in \quad) n = 20k' + 13$ $k \equiv 3[5]$ $7^{n+2} + n - 1 \equiv 0[5]$ $n = 4k+1$ 2

$(k' \in \quad) n = 20k' + 10$ $k \equiv 2[5]$ $7^{n+2} + n - 1 \equiv 0[5]$ $n = 4k+2$ 3

$(k' \in \quad) n = 20k' + 19$ $k \equiv 4[5]$ $7^{n+2} + n - 1 \equiv 0[5]$ $n = 4k+3$ 4

2 2017 60

$u_n = \frac{1}{3}(4^n - 1)$ n 1

$u_{n+1} = 4u_n + 1$ $u_0 = 0$ (u_n)

n $u_n = \frac{1}{3}(4^n - 1)$

P(0)

$u_0 = 0$ $u_0 = \frac{1}{3}(4^0 - 1) = 0$ n=0

$u_n = \frac{1}{3}(4^n - 1)$ P(n)

$u_{n+1} = \frac{1}{3}(4^{n+1} - 1)$ P(n+1)

$u_n = \frac{1}{3}(4^n - 1)$ $u_{n+1} = 4u_n + 1$

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$u_{n+1}=\frac{4}{3}.4^n-\frac{4}{3}+1=\frac{1}{3}.4^{n+1}-\frac{1}{3}=\frac{1}{3}(4^{n+1}-1)$ $u_{n+1}=4\left(\frac{1}{3}(4^n-1)\right)+1$

n $u_n=\frac{1}{3}(4^n-1)$

u_{n+1} u_n n

$ax_0+by_0=1$ $(x_0;y_0)\in$ ² **b a**

$u_{n+1}-4u_n=1$ $u_{n+1}=4u_n+1$

u_{n+1} u_n $u_{n+1}-4u_n=1$ $(1;-4)$

v_0 (v_n) **2**

$v_{n+1}=q.v_n$ n q (v_n)

$v_n=u_n+\frac{1}{3}$ n (v_n)

$v_{n+1}=u_{n+1}+\frac{1}{3}=4u_n+1+\frac{1}{3}=4u_n+\frac{4}{3}=4(u_n+\frac{1}{3})=4v_n$ $v_n=u_n+\frac{1}{3}$

$v_0=u_0+\frac{1}{3}=\frac{1}{3}$ $q=4$ (v_n)

S_n n

$S_n=v_0+v_1+v_2.....+v_{3n}$

$S_n=v_0\left[\frac{q^{3n+1}-1}{q-1}\right]=\frac{1}{3}\left[\frac{(4)^{3n+1}-1}{4-1}\right]=\frac{1}{9}\left(4^{3n+1}-1\right)$

$4^{n+1}-1$ 4^n-1 **3**

$3u_{n+1}=4^{n+1}-1$ $3u_n=4^n-1$ $u_n=\frac{1}{3}(4^n-1)$

$PGCD(3u_{n+1};3u_n)=3PGCD(u_{n+1};u_n)$

1 $PGCD(u_{n+1};u_n)=1$ $PGCD(4^{n+1}-1;4^n-1)=3.1=3$

7 4^n n **4**

$4^3\equiv 1[7]$ $4^2\equiv 2[7]$ $4^1\equiv 4[7]$ $4^0\equiv 1[7]$

3 7 4^n

$n=$	$3k$	$3k+1$	$3k+2$
$4^n\equiv$	1	4	2

A_n n

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$A_n = 9S_n - 6n - 3^{6n+4}$

$A_n \equiv 0[7]$ 7 A_n

$9S_n - 6n - 3^{6n+4} \equiv 0[7]$ $A_n \equiv 0[7]$

$4^{3n+1} \equiv 4[7]$ $9S_n = 3[7]$ $9S_n = \left(4^{3n+1} - 1\right)$

$3^{6n+4} \equiv \left[(4)^{3n+2}\right]^2 \equiv 4[7]$ $3^{6n+4} \equiv \left[(-4)^2\right]^{3n+2}[7]$

$6n \equiv 6[7]$ $3 - 6n - 4 \equiv 0[7]$ $9S_n - 6n - 3^{6n+4} \equiv 0[7]$

$p \in$ $n = 7p + 1$ 7 6 $n \equiv 1[7]$

1 2016 61

q u_2 u_1 **1**

$\begin{cases} \ln(u_1) + \ln(u_2) = 11 \\ u_1 + u_2 = e^4(1 + e^3) \end{cases}$ (u_n)

$\begin{cases} (u_1).(u_2) = e^{11} \\ u_1 + u_2 = e^4(1 + e^3) \end{cases}$ $\begin{cases} \ln(u_1).(u_2) = 11 \\ u_1 + u_2 = e^4(1 + e^3) \end{cases}$ $\begin{cases} \ln(u_1) + \ln(u_2) = 11 \\ u_1 + u_2 = e^4(1 + e^3) \end{cases}$

$P = e^{11}$ $S = e^4(1 + e^3)$ $x^2 - Sx + P = 0$ u_2 u_1

$x' = u_2 = e^7$ $x = u_1 = e^4$ $x^2 - Sx + P = 0$

$q = \frac{u_2}{u_1} = e^3$ $u_2 = q.u_1$

n u_n **2**

$u_n = e^4(e^3)^{n-1}) = e^{3n+1}$ $u_n = u_1.q^{n-1}$

n S_n

$S_n = \ln(u_0) + \ln(u_1) + ... + \ln(u_n)$

$S_n = \ln(u_0).(u_1).....(u_n) = \ln\left[(u_{0.})^{n+1}.q^{\frac{n(n+1)}{2}}\right] = (n+1)\ln((u_{0.}).q^{\frac{n}{2}})$

$S_n = (n+1)\ln((e).e^{\frac{3n}{2}}) = \frac{(n+1)(3n+2)}{2}$ $q = e^3$ $u_0 = e$

$PGCD(2S_n; a_n) = PGCD(a_n; 14)$ **n** **3**

$2S_n = (n+1)(3n+2) = 3n^2 + 5n + 2$ $a_n = n + 3$

$2S_n - a_n(3n-4) = 14$ $2S_n = (n+3)(3n-4) + 14 = a_n(3n-4) + 14$

$PGCD(2S_n; a_n) = PGCD(a_n; 14)$ n

$PGCD(2S_n; a_n)$

$PGCD(2S_n; a_n) = PGCD(a_n; 14)$

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14 7 2 1 14 $PGCD(2S_n;a_n)$

$PGCD(2S_n;a_n)=7$

$n\in$ $n=14k+4$ $PGCD(a_n;14)=7$ $PGCD(2S_n;a_n)=7$

7 2^n 4

7 2^n $2^3\equiv 1[7]$ $2^2\equiv 4[7]$ $2^1\equiv 2[7]$ $2^0\equiv 1[7]$

3

$n=$	$3k$	$3k+1$	$3k+2$
$2^n\equiv$	1	2	4

$\begin{cases} b_n\equiv 0[7] \\ n\equiv 0[5] \end{cases}$ n 5

$b_n=3n.a_n-2S_n+1437^{2016}+1=3n^2+9-(3n^2+5n+2)+1437^{2016}+1$

$2016\equiv 1[3]$ $1437\equiv 2[7]$ $1437^{2016}\equiv 2^{2016}[7]\equiv 1[7]$

$n\equiv 6[7]$ $15n\equiv 6[7]$ $5n-9\equiv 0[7]$ $b_n\equiv -5n+9[7]$

$2n\equiv 0[35]$ $\begin{cases} 5n\equiv 0[35] \\ 7n\equiv 0[35] \end{cases}$ $\begin{cases} n\equiv 0[7] \\ n\equiv 0[5] \end{cases}$ $\begin{cases} b_n\equiv 0[7] \\ n\equiv 0[5] \end{cases}$

k' $n=35k'$ $n\equiv 0[35]$

7 $1437^{9n+1}-3\times 4^{12n+1}+52$ n 6

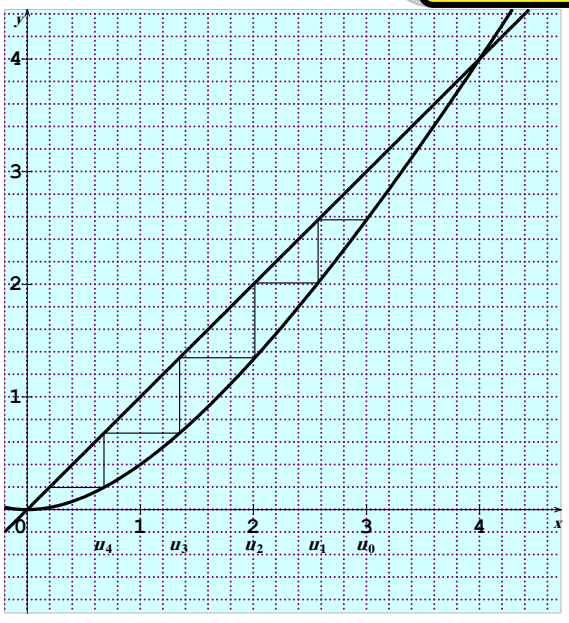
$1437^{9n+1}-3\times 4^{12n+1}+52\equiv 0[7]$ 7 $1437^{9n+1}-3\times 4^{12n+1}+52$

$52\equiv 3[7]$ $4^{12n+1}\equiv (4^{3n})^4\times 4\equiv 4[7]$ $1437^{9n+1}\equiv (2^{3n})^3.2\equiv 2[7]$

$1437^{9n+1}-3\times 4^{12n+1}+52\equiv -7[7]$ $1437^{9n+1}-3\times 4^{12n+1}+52\equiv 2-3.4+3[7]$

$1437^{9n+1}-3\times 4^{12n+1}+52\equiv 0[7]$

1 2014 62



f (1

$f(x)=\dfrac{2x^2}{x+4}$ $[0;+\infty[$ f

f

$x\in[0;+\infty[$ $f'(x)$ 0 $x\in[0;+\infty[$

$f'(x)=\dfrac{4x(x+4)-2x^2}{(x+4)^2}=\dfrac{2x(x+8)}{(x+4)^2}$

f $x\in[0;+\infty[$ $f'(x)$ 0

u_4 u_3 u_2 u_1 u_0 2

(u_n)

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0

$0 \le u_n \le 3$ $n \in$ 3

P(0) •

$u_0 = 3$ $0 \le u_0 \le 3$ n=0

$0 \le u_n \le 3$ P(n)

$0 \le u_{n+1} \le 3$ P(n+1)

$0 \le u_{n+1} \le \frac{18}{7}$ **f** $f(0) \le f(u_n) \le f(3)$ $0 \le u_n \le 3$

$0 \le u_n \le 3$ $\frac{18}{7}$ 3 $0 \le u_{n+1} \le 3$

(u_n)

n $u_{n+1} \le u_n$ (u_n)

$$u_{n+1} - u_n = \frac{2u_n^2}{u_n + 4} - u_n = \frac{u^2_n - 4u_n}{u_n + 4}$$

$u^2_n - 4u_n$

$-u^2_n + 2u_n + 8$ $u_n = 0$ $u_n = 4$ $u^2_n - 4u_n = 0$

u_n	0	4	$+\infty$
	0		
	(u_n)		(u_n)

(u_n) $0 \le u_n \le 3$

(u_n)

3 3 (u_n)

$7u_{n+1} - 6u_n$ 4

$$7u_{n+1} - 6u_n = \frac{14u_n^2}{u_n + 4} - 6u_n = \frac{8u_n(u_n - 3)}{u_n + 4} =$$

$u_n(u_n - 3)$ $7u_{n+1} - 6u_n$

$7u_{n+1} - 6u_n$ $u_n = 0$ $u_n = 3$ $u_n(u_n - 3) = 0$

u_n	0	3	$+\infty$
$7u_{n+1} - 6u_n$		0	

$0 \le u_n \le 3$ $7u_{n+1} - 6u_n$ 0

$0 \le u_{n+1} \le \frac{6}{7}u_n$ n

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2 $0 \le u_{n+1} \le 3$ 1 u_{n+1} $\frac{6}{7}u_n$ $7u_{n+1} - 6u_n$ 0

$0 \le u_{n+1} \le \frac{6}{7}u_n$ 2 1

$0 \le u_n \le 3\left(\frac{6}{7}\right)^n$ n

P(0) •

$u_0 = 3$ $0 \le u_0 \le 3\left(\frac{6}{7}\right)^0 = 3$ n=0

$0 \le u_{n+1} \le 3\left(\frac{6}{7}\right)^{n+1}$ P(n+1) $0 \le u_n \le 3\left(\frac{6}{7}\right)^n$ P(n)

$\frac{6}{7}$ $0 \le \frac{6}{7}u_n \le 3\left(\frac{6}{7}\right)^{n+1}$ $0 \le u_n \le 3\left(\frac{6}{7}\right)^n$

$0 \le u_{n+1} \le 3\left(\frac{6}{7}\right)^{n+1}$ $0 \le u_{n+1} \le \frac{6}{7}u_n$

n $0 \le u_n \le 3\left(\frac{6}{7}\right)^n$

$+\infty$ n (u_n)

$\lim\limits_{n\to+\infty} u_n = 0$ $\lim\limits_{n\to+\infty}\left(\frac{6}{7}\right)^n = 0$ $0 \le u_n \le 3\left(\frac{6}{7}\right)^n$

2 2012 63

$u_{n+1} = 6u_n - 9$ n $u_0 = 16$ (u_n)

7 u_4 u_3 u_2 u_1 u_0 1

$u_{n+1} = 6u_n - 9$ $u_0 = 16$

$u_4 = 6u_3 - 9 \equiv 3[7]$ $u_2 = 6u_1 - 9 \equiv 2[7]$ $u_1 = 6u_0 - 9 \equiv 3[7]$ $u_0 = 16 \equiv 2[7]$

$u_{2k+1} \equiv b[7]$ $u_{2k} \equiv a[7]$ b a

$u_{2k+1} \equiv 3[7]$ $u_{2k} \equiv 2[7]$

$u_{n+2} \equiv u_n[7]$ n 2

P(0) •

$u_0 \equiv 2[7]$ $u_2 \equiv 2[7]$ $u_2 \equiv u_0[7]$ n=0

$u_{n+2} \equiv u_n[7]$ P(n)

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$u_{n+3} \equiv u_{n+1}[7]$ P(n+1)

$u_{n+3} = 6u_{n+2} - 9 = 6(6u_{n+1} - 9) - 9 = 36u_{n+1} - 63$ $u_{n+1} = 6u_n - 9$

$u_{n+3} \equiv u_{n+1}[7]$ $63 \equiv 0[7]$ $36 \equiv 1[7]$

$u_{2k} \equiv 2[7]$ k

P(0) •

$u_0 \equiv 2[7]$ k=0

$u_{2k+2} \equiv 2[7]$ P(k+1) $u_{2k} \equiv 2[7]$ P(k)

n $u_{n+2} \equiv u_n[7]$

n=2k $u_{2k+2} \equiv u_{2k}[7]$

$u_{2k+2} \equiv 2[7]$ $u_{2k} \equiv 2[7]$

k $u_{2k} \equiv 2[7]$

$u_{2k+1} \equiv 3[7]$

$u_{2k+1} = 6u_{2k} - 9$ $u_{n+1} = 6u_n - 9$

$u_{2k+1} \equiv 3[7]$ $u_{2k} \equiv 2[7]$ $u_{2k+1} \equiv 6(2) - 9[7]$

(v_n) 3

$v_n = u_n - \frac{9}{5}$ n

$v_{n+1} = q.v_n$ n (v_n)

$v_{n+1} = u_{n+1} - \frac{9}{5} = 6u_n - 9 - \frac{9}{5} = 6u_n - \frac{54}{5} = 6(u_n - \frac{9}{5}) = 6v_n$ $v_n = u_n - \frac{9}{5}$

$v_0 = u_0 - \frac{9}{5} = 16 - \frac{9}{5} = \frac{71}{5}$ 6 (v_n)

$S_n = u_0 + u_1 + ... + u_n$ S_n u_n n

u_n n

$u_n = v_n + \frac{9}{5} = q^n.v_0 + \frac{9}{5} = \frac{71}{5}.(6)^n + \frac{9}{5}$ $v_n = u_n - \frac{9}{5}$

S_n n

$S_n = u_0 + u_1 + ... + u_n = (v_0 + \frac{9}{5}) + (v_1 + \frac{9}{5}) + ... + (v_n + \frac{9}{5}) = (v_0 + v_1 + ... + v_n) + \frac{9}{5}(n+1)$

$(v_0 + v_1 + ... + v_n) = v_0\left[\frac{q^{n+1} - 1}{q - 1}\right]$

$S_n = = \frac{71}{5}\left[\frac{6^{n+1} - 1}{5 - 1}\right] + \frac{9}{5}(n+1) = 71\left[\frac{6^{n+1} - 1}{25}\right] + \frac{9}{5}(n+1)$

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u_5 u_3 1

(u_n)

$\begin{cases} m = PPCM(u_3;u_5) \\ d = PGCD(u_3;u_5) \end{cases}$ $\begin{cases} u_4 = 15 \\ m + d = 42 \end{cases}$

$PGCD(u'_3;u'_5) = 1$ $u_5 = d.u'_5$ $u_3 = d.u'_3$ $d = PGCD(u_3;u_5)$

$m = d.u'_3.u'_5$ $m.d = u_3.u_5$

$u'_3.u'_5 + 1 = \frac{42}{d}....(1)$ $m + d = 42$

$u_5 + u_3 = 2u_4$ u_5 u_4 u_3

$u'_3 + u'_5 = \frac{30}{d}....(2)$ $u_5 + u_3 = 2u_4$

$d \in D_6 = \{1;2;3;6\}$ 42 30 d 2 1

$d = 6$ 2 1

$u'_5 = 3$ $u'_3 = 2$ $u'_3 + u'_5 = 5$ $u'_3.u'_5 = 6$ $d = 6$

$u_5 = 18$ $u_3 = 12$

u_0

$u_0 = u_3 - 3r = 3$ $u_4 - u_3 = r = 3$ $u_3 = u_0 + 3r$ (u_n)

n u_n 2

$u_n = 3 + 3n$ $u_n = u_0 + n.r$ n

(u_n) 2010

$u_n = 2010$ n (u_n) 2010

$n = 669$ $2010 = 3n + 3$ $u_n = 2010$

670 2010 $u_{669} = 2010$ $n = 669$

10080 (u_n) 5 3

u_p

$\frac{5}{2}(u_p + u_{p+4}) = 10080$ 10080 (u_n) 5

$\frac{5}{2}(2u_p + 4r) = 10080$ $\frac{5}{2}(u_p + u_{p+4}) = 10080$ $u_{p+4} = u_p + 4r$

$u_p = 2010$ $r = 3$ $u_p = \frac{1}{2}\left[\frac{2}{5}(10080) - 4r\right]$

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$S = u_0 + u_1 + \ldots + u_{2n}$ S n 4

$$S = u_0 + u_1 + \ldots + u_{2n} = \frac{2n+1}{2}(u_0 + u_{2n}) = \frac{2n+1}{2}(6n+6) = 3(2n+1)(n+1)$$

S_2 S_1 n

$$S_1 = u_0 + u_2 + u_4 \ldots + u_{2n} = u_0 + (u_0 + 2r) + (u_0 + 4r) \ldots + (u_0 + 2nr)$$

2r u_0 S_1

$$S_2 = u_1 + u_3 + u_4 \ldots + u_{2n-1} = S - S_1 = 3n(n+1) \qquad S_1 = \frac{n+1}{2}(2u_0 + 2nr) = 3(n+1)^2$$

1 2009 65

f 1

$f(x) = \frac{1}{2}(x + \frac{5}{x})$ $[1,5]$ f

$f'(x) = \frac{1}{2}(1 - \frac{5}{x^2}) = \frac{x^2-5}{2x^2}$ $[1,5]$ x

$x = -\sqrt{5}$ $x = \sqrt{5}$ $x^2 - 5 = 0$ $f'(x) = 0$

$x \in [\sqrt{5};5[$ $f'(x)$ 0 $x \in [1;\sqrt{5}[$ $f'(x)$ 0

x	1	$\sqrt{5}$	5
f'(x)		0	
f(x)	3	$\sqrt{5}$	3

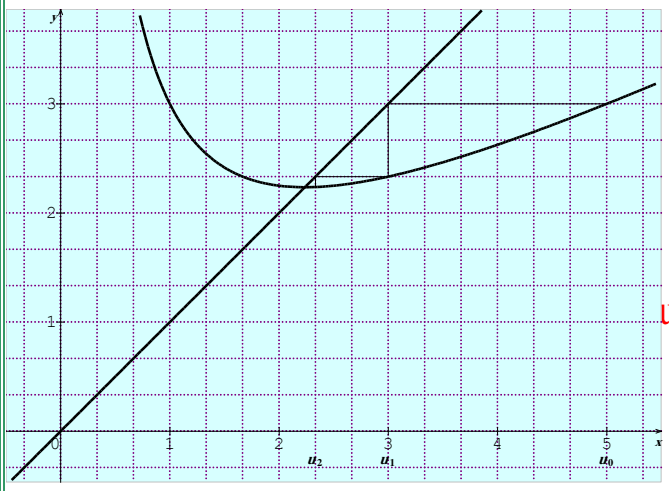
$y = x$ (Δ) (C)

u_2 u_1 2

$u_0 = 5$ $u_{n+1} = \frac{1}{2}(u_n + \frac{5}{u_n})$ (u_n)

$u_2 = \frac{1}{2}(u_1 + \frac{5}{u_1}) = \frac{7}{3}$ $u_1 = \frac{1}{2}(u_0 + \frac{5}{u_0}) = 3$

u_2 u_1 u_0



$u_n \geq \sqrt{5}$ n 3

P(0) •

$u_0 = 5$ $u_0 \geq \sqrt{5}$ n=0

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$u_n \geq \sqrt{5}$ P(n)

$u_{n+1} \geq \sqrt{5}$ P(n+1)

$[\sqrt{5};5[$ f $f(u_n) \geq f(\sqrt{5})$ $u_n \geq \sqrt{5}$

$f(\sqrt{5}) = \sqrt{5}$ $f(u_n) = u_{n+1}$ $u_{n+1} \geq \sqrt{5}$ $f(u_n) \geq f(\sqrt{5})$

n $u_n \geq \sqrt{5}$

(u_n)

n $u_{n+1} - u_n$ 0 (u_n)

$$u_{n+1} - u_n = \frac{1}{2}(u_n + \frac{5}{u_n}) - u_n = \frac{1}{2}(-u_n + \frac{5}{u_n}) = \frac{1}{2}(\frac{-u^2{}_n + 5}{u_n})$$

(u_n) $u_n \geq \sqrt{5}$ $\frac{1}{2}(\frac{-u^2{}_n + 5}{u_n})$ 0

$(u_{n+1} - \sqrt{5}) \leq \frac{1}{2}(u_n - \sqrt{5})$ n $\in$ 4

$(u_{n+1} - \sqrt{5}) - \frac{1}{2}(u_n - \sqrt{5}) \leq 0$ $(u_{n+1} - \sqrt{5}) \leq \frac{1}{2}(u_n - \sqrt{5})$

$$(u_{n+1} - \sqrt{5}) - \frac{1}{2}(u_n - \sqrt{5}) = \frac{1}{2}(u_n + \frac{5}{u_n}) - \frac{1}{2}u_n - \frac{\sqrt{5}}{2} = \frac{5}{2u_n} - \frac{\sqrt{5}}{2} = \frac{\sqrt{5}(\sqrt{5} - u_n)}{2u_n}$$

$(u_{n+1} - \sqrt{5}) - \frac{1}{2}(u_n - \sqrt{5}) \leq 0$ $\frac{\sqrt{5}(\sqrt{5} - u_n)}{2u_n} \leq 0$ $u_n \geq \sqrt{5}$

$\lim\limits_{n \to +\infty} u_n$ $(u_n - \sqrt{5}) \leq \left(\frac{1}{2}\right)^n (u_0 - \sqrt{5})$

$(u_{n+1} - \sqrt{5}) \leq \frac{1}{2}(u_n - \sqrt{5})$ n $\in$

$(u_1 - \sqrt{5}) \leq \frac{1}{2}(u_0 - \sqrt{5})$ n=0

$(u_3 - \sqrt{5}) \leq \frac{1}{2}(u_2 - \sqrt{5})$ n=3 $(u_2 - \sqrt{5}) \leq \frac{1}{2}(u_1 - \sqrt{5})$ n=1

$(u_{n-1} - \sqrt{5}) \leq \frac{1}{2}(u_{n-2} - \sqrt{5})$ n-2

$(u_n - \sqrt{5}) \leq \frac{1}{2}(u_{n-1} - \sqrt{5})$ n-1

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$$(u_n-\sqrt{5})\le\left(\frac{1}{2}\right)^n(u_0-\sqrt{5})$$

$$\lim_{n\to+\infty}\left(\frac{1}{2}\right)^n(u_0-\sqrt{5})=0 \qquad 0\le(u_n-\sqrt{5})\le\left(\frac{1}{2}\right)^n(u_0-\sqrt{5})$$

$$\lim_{n\to+\infty}u_n=\sqrt{5} \qquad \lim_{n\to+\infty}(u_n-\sqrt{5})=0$$

2 2009 66

v_n β α 1

q $v_{n+1}=q.v_n$ v_n

$v_{n+1}=u_{n+1}+\alpha(n+1)+\beta=3u_n+2n+1+\alpha n+\alpha+\beta$ $v_n=u_n+\alpha n+\beta$

$v_{n+1}=3u_n+2n+1+\alpha n+\alpha+\beta...(1)$

$u_n=v_n-\alpha n-\beta$ $v_n=u_n+\alpha n+\beta$

1 $u_n=v_n-\alpha n-\beta$

$v_{n+1}=3v_n-3\alpha n-3\beta+2n+1+\alpha n+\alpha+\beta=3v_n+2(1-\alpha)n-2\beta+1$

$-2(\alpha-1)-2\beta+\alpha+1=0$ v_n

$\beta=1$ $\alpha=1$ $-2\beta+\alpha+1=0$ $\alpha-1=0$

$v_{n+1}=3.v_n$ $\beta=1$ $\alpha=1$

$v_0=u_0+1.0+1=1$ 3

n u_n v_n 2

$v_n=v_0.q^n=1.(3)^n$ v_n

$u_n=v_n-n-1=(3)^n-n-1$

S' S 3

$$S=v_0+v_1+v_2+...+v_n=v_0\left[\frac{q^{n+1}-1}{q-1}\right]=1.\left[\frac{3^{n+1}-1}{3-1}\right]=\frac{3^{n+1}-1}{2}$$

$u_n=v_n-n-1$ $S'=u_0+u_1+u_2+...+u_n$

$S'=v_0+v_1+v_2+...+v_n-(0+1+2+....+n)-(n+1)=S-(n+1)(n+2)$

5 3^n n 4

$3^4\equiv1[5]$ $3^3\equiv2[5]$ $3^2\equiv4[5]$ $3^1\equiv3[5]$ $3^0\equiv1[5]$

4 5 3^n

n	4k	4k+1	4k+2	4k+3	
$3^n\equiv$	1	3	4	2	

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5 u_n n

$(3)^n - n \equiv 1[5]$ $(3)^n - n - 1 \equiv 0[5]$ 5 u_n

p $n = 20p$ $k = 5p$ $k \equiv 0[5]$ $-4k \equiv 0[5]$ $n = 4k$ 1

$n = 20p + 17$ $k \equiv 4[5]$ $n = 4k + 1$ 2

$n = 20p + 18$ $k \equiv 4[5]$ $n = 4k + 2$ 3

$n = 20p + 11$ $k \equiv 2[5]$ $n = 4k + 3$ 4

1 2008 67

$\lim\limits_{x \longrightarrow 1} \dfrac{f(x) - f(1)}{x - 1}$ 1

$f(x) = 3 + \sqrt{x - 1}$ $[1 : +\infty[$ f

$$\lim_{x \longrightarrow 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x \longrightarrow 1} \frac{\sqrt{x - 1}}{x - 1} = \lim_{x \longrightarrow 1} \frac{1}{\sqrt{x - 1}} = +\infty$$

1 f

$x = 1$ (C_f)

f

$\lim\limits_{x \to +\infty} f(x) = 3 + \lim\limits_{x \to +\infty} \sqrt{x - 1} = +\infty$ $\lim\limits_{x \longrightarrow 1} f(x) = f(1) = 3$

$f'(x) = \dfrac{1}{2\sqrt{x - 1}}$ $]1; +\infty[$

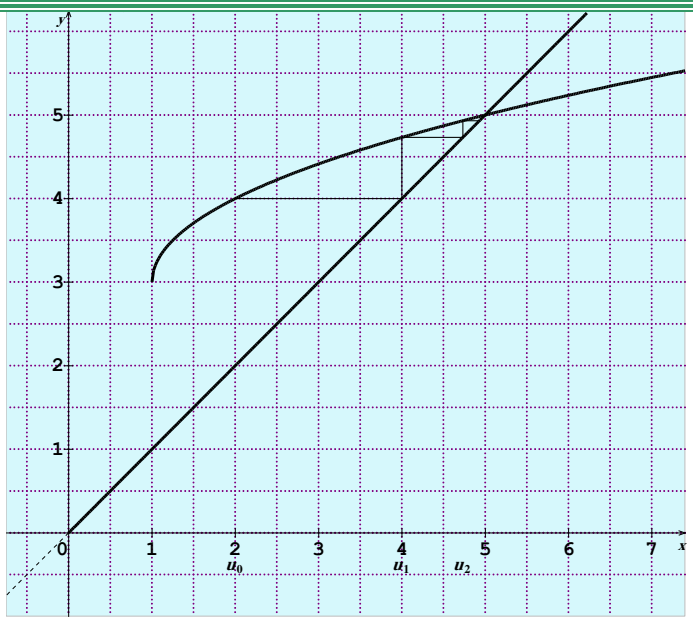
$f'(x)$ 0 $[1 : +\infty[$ f

x	1	$+\infty$
f'(x)		
f(x)	3	$+\infty$

(C_f)

$y = x$ (D)

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u_2 u_1 u_0 (C_f) (D) 2

(u_n)

5 (u_n)

$2 \le u_n \le 5$ $n \in$ * 3

$P(1)$ •

$u_1 = 3$ $2 \le u_1 \le 5$ n=1

$2 \le u_n \le 5$ P(n)

$2 \le u_{n+1} \le 5$ P(n+1)

$[1;+\infty[$ f $f(2) \le f(u_n) \le f(5)$ $2 \le u_n \le 5$

$f(5) = 5$ $f(2) = 4$ $f(u_n) = u_{n+1}$ $4 \le u_{n+1} \le 5$ $f(2) \le f(u_n) \le f(5)$

$[2;5]$ $[4;5]$ $2 \le u_{n+1} \le 5$ $4 \le u_{n+1} \le 5$

$2 \le u_n \le 5$

u_{n+1} u_n $n \in$ *

$P(1)$ •

$u_2 = f(u_1) = 4$ $u_1 = 3$ u_2 u_1 n=1

u_{n+1} u_n P(n)

u_{n+2} u_{n+1} P(n+1)

$[1;+\infty[$ f $f(u_{n+1})$ $f(u_n)$ u_{n+1} u_n

$f(u_{n+1}) = u_{n+2}$ $f(u_n) = u_{n+1}$ u_{n+2} u_{n+1} $f(u_{n+1})$ $f(u_n)$

u_{n+1} u_n

$\lim\limits_{n \to +\infty} u_n$ (u_n)

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(u_n) $n \in {}^*$ u_{n+1} u_n

5 (u_n) $n \in {}^*$ $2 \le u_n \le 5$

- $\lim\limits_{n\to+\infty} u_n = \alpha$ α

$3+\sqrt{\alpha-1}=\alpha$ $f(\alpha)=\alpha$ $f(u_n)=u_{n+1}$

$\alpha \ge 3$ $\alpha-1=(\alpha-3)^2$ $\sqrt{\alpha-1}=\alpha-3$ $3+\sqrt{\alpha-1}=\alpha$

$\lim\limits_{n\to+\infty} u_n = 5$ $\alpha=2$ $\alpha=5$ $\alpha^2-7\alpha+10=0$

1 2008 68

u_3 u_2 u_1 1

$u_0=2$ $u_{n+1}=\frac{2}{3}u_n+1$ (u_n)

$u_3=\frac{2}{3}u_2+1=\frac{81}{27}$ $u_2=\frac{2}{3}u_1+1=\frac{16}{9}$ $u_1=\frac{2}{3}u_0+1=\frac{7}{3}$

$\lim\limits_{n\to+\infty} u_n$ n u_n (v_n) 2

$v_n=u_n+\left(\frac{2}{3}\right)^n$ $n \in$ (v_n)

$n \in$ $v_{n+1}=v_n$ (v_n)

- P(0)

$v_1=u_1+\left(\frac{2}{3}\right)=v_0=u_0+\left(\frac{2}{3}\right)^0=3$ $v_1=v_0$ n=0

$n \in$ $v_{n+1}=v_n$ P(n)

$n \in$ $v_{n+2}=v_{n+1}$ P(n+1)

$v_{n+1}=u_{n+1}+\left(\frac{2}{3}\right)^{n+1}=\frac{2}{3}u_n+1+\left(\frac{2}{3}\right)^{n+1}$ $v_{n+2}=u_{n+2}+\left(\frac{2}{3}\right)^{n+2}=\frac{2}{3}u_{n+1}+1+\left(\frac{2}{3}\right)^{n+2}$

$v_{n+1}=v_n$ $v_{n+2}-v_{n+1}=\frac{2}{3}\left[u_{n+1}+\left(\frac{2}{3}\right)^{n+1}\right]-\frac{2}{3}\left[u_n+\left(\frac{2}{3}\right)^n\right]=\frac{2}{3}(v_{n+1}-v_n)=0$

$\lim\limits_{n\to+\infty} u_n=\lim\limits_{n\to+\infty} v_n-\lim\limits_{n\to+\infty}\left(\frac{2}{3}\right)^n=v_0=3$ $u_n=v_n-\left(\frac{2}{3}\right)^n$ $v_n=u_n+\left(\frac{2}{3}\right)^n$

$\lim\limits_{n\to+\infty}\left(\frac{2}{3}\right)^n=0$ (v_n) $\lim\limits_{n\to+\infty} v_n=v_0=3$

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$$w_n = \frac{2}{3}n - \left(\frac{2}{3}\right)^n : n \in \qquad (w_n)$$

$$S = w_0 + w_1 + w_2 + ... + w_n = \frac{2}{3}(0+1+2+.....+n) - \left[\left(\frac{2}{3}\right)^0 + \left(\frac{2}{3}\right)^1 + ... + \left(\frac{2}{3}\right)^n\right]$$

$$(0+1+2+.....+n) = \frac{n(n+1)}{2}$$

$$\left(\frac{2}{3}\right)^0 + \left(\frac{2}{3}\right)^1 + ... + \left(\frac{2}{3}\right)^n = 1.\left[\frac{\left(\frac{2}{3}\right)^{n+1} - 1}{\left(\frac{2}{3}\right) - 1}\right] = -3\left[\left(\frac{2}{3}\right)^{n+1} - 1\right]$$

$$S = \frac{1}{3}\left[n^2 + n + 6\left(\frac{2}{3}\right)^{n+1} - 3\right]$$

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