****



$$2020 \equiv \ldots [1440]$$

****

$$PGCD(2020;1440) = \ldots$$

BAC2020

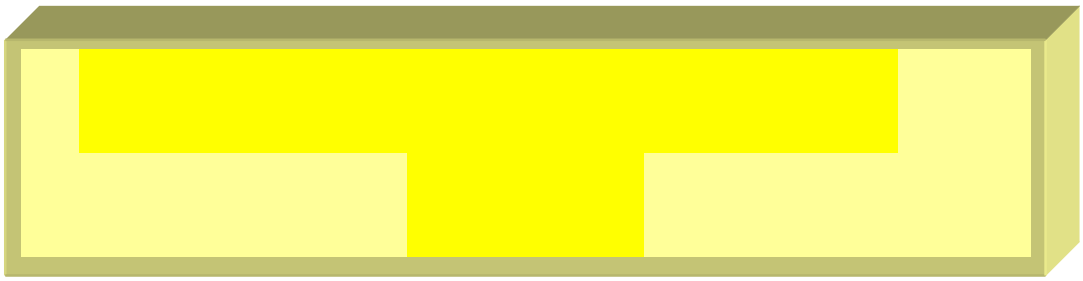
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2 ، 1

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01

75 (1

$ab = 75$ (a,b) 2

$(x-1)(y+1) = 75$ $(x;y)$

02

$(x;y)$ 2

$5xy - y^2 = 49$ 4 $xy = 3x + 2y$ 3 $x^2 = 4y^2 + 3$ 2 $x^2 - y^2 = 15$ 1

03

n

$n+8$ $5n+6$ 3 12 $5n+7$ 2 $|n| \le 22$ $n+4$ 13 1

04

276 1440 d 1

276 1440 2

3

1440u 276v d v u

05

37 n 21685 n

n 53 n 33509

06

a,b

$2a^2$ b^2 2112 3 a^2 b^2 600 3 $\begin{cases} ab = 360 \\ PGCD(a,b) = 6 \end{cases}$ 2 a b 72

$PGCD\ a,b$ 8 $PGCD\ a,b$ 5 $PGCD\ a,b$ 9 1

07

$a = n(n^2+5)$ n

3 a 2 a 1

6 a

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08

k 4 0 2^k 5

k 2^k 5 **1**

5 17^{4k} **2**

5 $2^{4k+3}+17^{4k+2}+3$ **3**

5 $2007^{1999}-61^{2008}+87^{49}$

09

7 6^{2n} 1 n

7 5^n n 2

7 A_n 3 6^{2n} 5^n n 3

10

9 7^n n 1

9 7^n $3n$ 1 n 2

11

7 3^{2n-2} 2^{n-1} n 1

11 2^{2n-1} 3^{n-2} 1 n 2

n n^2 1 n 3

12

5 x^2 x

y x $x^2-5y^2=3$

13

7 $2n^3$ n 2 n 1

n^2 $3n$ 6 0 11 n (2

121 n^2 $3n$ 6 n 3

14

7^n 2^n n 1

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3548^{9} 2537^{31}

5^{4n} 5^{3n} 5^{2n} 5^{n} 0 13 n 2

15

7 3^{n} 1

n 6 5 4 3 2 1

n 7 3^{n} 2

$\left(3^{2020}+10^{1440}+9^{3n+2}\right)$ 7 3

16

$\overline{1101101}$ n 1

$\overline{214}$ n

$\overline{51}=\overline{13}+\overline{35}$ 2

17

$45x-28y=130$ (x,y) (

5 y x

$\overline{5\beta\beta6}$ 9 $\overline{2\alpha\alpha3}$ n

7

18

$n=\overline{1271x}$ n 9

8 n x 1

11 n x 2

19

y x (1

7 $\overline{xy}$ $\overline{yx}$

20

$b=\overline{102}^{(3)}$ $a=\overline{413}^{(5)}$ 1

b a

$a\times b$ $a+b$ 7

x -2

$\overline{xxx}^{(9)}=\overline{52\alpha}^{(11)}$ $\overline{12}^{(x)}\times\overline{34}^{(x)}=\overline{452}^{(x)}$

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21

$41x \quad 27y \quad 1 \quad ^{2} \quad x, y \quad 1 \quad 1$

1 (أ

1 (ب

$2 \quad ^{2} \quad 41x \quad 27y \quad 5 \ldots\ldots 2 \quad 2$

22

$(1) \ldots 2045x - 64y = 1 \quad (x, y) \quad ^{2}$

$^{2} \quad (1) \quad PGCD(2045,64) \quad 1$

$(1) \quad ^{2} \quad (1) \quad 2$

23

$x, y \quad S$

$11x \quad 3y \quad 65 \ldots\ldots\ldots\ldots 1$

$1 \quad ^{2} \quad 2x_0^{2} \quad 3y_0 \quad 11 \quad S \quad x, y \quad 1$

$y \quad 5 \quad x \quad 5 \quad S \quad x, y \quad 2$

24

(x, y)

$x \le y \quad \begin{cases} ppcm(x, y) = 12p \gcd(x, y) \\ x + y = 105 \end{cases} \quad 2 \quad x \le y \quad \begin{cases} ppcm(x, y) = 60 \\ xy = 180 \end{cases} \quad 1$

$ppcm(x, y) \quad 100 \quad 4 \quad \begin{cases} 3ppcm(x, y) = xy \\ x^2 - y^2 = 405 \end{cases} \quad 3$

$p \gcd(x, y) \quad 5$

25

$2n \quad 3 \quad n \quad 1 \quad n \quad 1$

$3n \quad 4 \quad n \quad 1$

$6n^2 \quad 17n \quad 12 \quad n \quad 1$

$b = n + 2 \quad a = 2n^2 + 4n + 1 \quad n \quad 2$

$b \quad a$

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2019 26

y x $5x-3y=1...(E)$ $(x;y)$ 1

n (E) $(6n+2;10n+3)$

$6n+2$ $10n+3$

b a d $b=3n+5$ $a=10n+3$ 2

$d=41$ $d=1$

$n\equiv 12[41]$ $d=41$

$B=6n^2+19n+15$ $A=20n^2+36n+9$ 3

$2n+3$ B A

B A n n

2017 27

$4^{5k}\equiv 1[11]$ k 1

11 4^n n 2

11 $(2\times 2017^{5n+3}+3\times 1438^{10n}+1)$ n 3

11 $(2\times 2017^{5n+2}+n-3)$ n 4

2017 28

5 3^n n 1

5 1437^{2017} 2

5 $(48^{4n+3}-2\times 9^{2n+1}+1)$ n 3

5 $(3^{4n}+27^n-4)$ n 4

1 2016 29

y x $6x-7y=19$ $(x;y)$ (E)

(E) $x_0=y_0$ (E) $(x_0;y_0)$ 1

42 λ $\begin{cases}\lambda\equiv 24[7]\\ \lambda\equiv 5[6]\end{cases}$ λ 2

$|x+y-1|\leq 13$ (E) $(x;y)$ 3

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7 5^n 4

$\begin{cases} n - 5^n \equiv 2020[7] \\ n \equiv 1437[6] \end{cases}$ n

1 2015 30

13 8^n العدد الطبيعي n ، (أ-1

13 $42 \times 138^{2015} + 2014^{2007} - 3$:

$(5n+1) \times 64^n - 5^{2n+3} \equiv (5n+6)8^{2n}[13]$ n (أ-2

$(5n+1) \times 64^n - 5^{2n+3} \equiv 0[13]$ n (

1 2013 31

$11x + 7y = 1$ $(x;y)$ (E) y x

$x_0 + y_0 = -1$ (E) $(x_0;y_0)$ 1

(E)

$\begin{cases} S = 11a + 1 \\ S = 7b + 2 \end{cases}$ S b a 2

(E) $(a;-b)$

77 S

1 2012 32

11 9^n n 1

11 2011^{2012} 2

11 $4 \times 9^{15n+1} + 4 \times 2011^{10n} + 2011^{2012}$ n 3

11 $2011^{2012} + 2n + 2$ n 4

2 2012 33

x $\begin{cases} x \equiv 3[15] \\ x \equiv 6[7] \end{cases}$ (S)

(S) 153 1

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$\left(\begin{cases} x - x_0 \equiv 0[15] \\ x - x_0 \equiv 0[7] \end{cases}\right)$ (S) x (S) x_0 2

(S) 3

3 15 4

6 7

600 500

2 2011 34

$A_n = 2^n + 3^n + 4^n + 5^n + 6^n$ n

$A_3 \equiv 6[7]$ $4 \equiv -3[7]$ 1

7 3^n 2^n n 2

7 $A_n + 1$ n 3

7 A_{2011}

7 A_{1432} 4

1 2010 35

7 n

α $n = \overline{11\alpha 00}$

3 n α 1

5 n α 2

15 n α

n $\alpha = 4$ 3

2 2010 36

13 10^n n 1

$(10^{2008})^2 + 10^{2008} + 1 \equiv 0[13]$ 2

$10^{2n} + 10^n + 1 \equiv 0[13]$ n 3

2 2009 37

$y' = (\ln 2)y$ 1

$f(x)$ $f(0) = 1$ f 2

2^n 7 n 3

$f(2009)-4$ 7

$S_n = f(0)+f(1)+...f(n)$ n 4

7 S_n n

1 2008 38

5 n

$b = 2n+3$ $a = n-2$ b a 1

b a

7 $n+5$ 7 b a

$PGCD(a;b) = 7$ n

$q = n^2-7n+10$ $p = 2n^2-7n-15$ q p 2

$n-5$ q p

$PGCD(p;q)$ n n

2 2008 39

$4x-9y = 319....(1)$ y x

1 (1) $(82;1)$ 1

$4a^2-9b^2 = 319....(2)$ $(a;b)$ 2

y_0 x_0 1 $(x_0;y_0)$

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2018 40

$\begin{cases}\alpha+\beta=4035\\ \alpha-\beta=1\end{cases}$ β α 1

β $\frac{\alpha}{2}$ β α

. $1009x-2017y=1$ $(x;y)$ 2

$\begin{cases}a\equiv 2019[2017]\\ a\equiv 2019[1009]\end{cases}$ a 3

9 7^n n n 4

$L=\overline{111.....1}$ 7 L

2018 مر

9 42L

1 2017 41

y x $(x;y)$ $104x-20y=272....(E)$ 1

(E) 20 104

$x\equiv 3[5]$ (E) $(x;y)$

$\overline{1\alpha\beta01}$ 4 $\overline{1\alpha\alpha\beta01}$ λ 2 (E)

λ β α β α 6

$(a;b)$ 1009 2017 3

$m=PPCM(a;b)$ $d=PGCD(a;b)$ $2m-d=2017$

2 2017 42

$u_0=1$ (u_n)

$u_{n+1}=7u_n+8:n$

$3u_n=7^{n+1}-4$ n 1

$S_n'=u_0+u_1+.....+u_n$ $S_n=1+7+7^2.....+7^n$ n 2

S'_n S_n S_n n

$18\times S'_n=7^{n+2}-24n-31$ n

5 7^n n 3

5 S'_n n

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2017 43

$63x+5y=159...(E)$ y x (E)

(E) 63 5 1

(E) $x\equiv 3[5]$ (E) $(x;y)$ 2

5 $\overline{\beta 10\beta 0}$ 7 $\overline{5\alpha 0\alpha}$ λ 3

$\lambda+2$ β α

5 3^n n 4

5 $3^{x-y}+4n+1438^{2017}$ n

x (E) $(x;y)$

2017 44

$u_0=0$ u_0 (u_n)

$u_{n+1}=4u_n+1$

$u_n=\frac{1}{3}(4^n-1)$ n 1

u_{n+1} u_n n

$v_n=u_n+\frac{1}{3}$ n (v_n) 2

v_0 (v_n)

$S_n=v_0+v_1+v_2.....+v_{3n}$ S_n n

$4^{n+1}-1$ 4^n-1 n 3

7 4^n n 4

7 $A_n=9S_n-6n-3^{6n+4}$ A_n n

1 2016 45

$\begin{cases}\ln(u_1)+\ln(u_2)=11\\ u_1+u_2=e^4(1+e^3)\end{cases}$ q u_0 (u_n)

q u_2 u_1 1

n u_n $q=e^3$ $u_1=e^4$ 2

n S_n $S_n=\ln(u_0)+\ln(u_1)+.....+\ln(u_n)$

$a_n=n+3$ n 3

$PGCD(2S_n;a_n)=PGCD(a_n;14)$ 3

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$PGCD(2S_n;a_n)$

$PGCD(2S_n;a_n)=7$

7 2^n 4

$b_n=3n.a_n-2S_n+1437^{2016}+1$ 5

$\begin{cases} b_n\equiv 0[7] \\ n\equiv 0[5] \end{cases}$ n

7 $1437^{9n+1}-3\times 4^{12n+1}+52$ n 6

2 2016 46

11 7^n 3^n 1

11 $2\times 2016^{5n+4}+1437^{10n+4}$ n

y x $7x-3y=8$ $(x;y)$ (E) 2

(E)

(E) $(x;y)$ y x d

d

$d=4$ (E) $(x;y)$

$2016^{7x}+1437^{3y}\equiv 0[11]$ (E) $(x;y)$

1 2015 47

7 2^n العدد الطبيعي n ، (أ- 1

7 $1962^{1954}+1954^{1962}+2015^{53}$:

. 7832 89 (أ-2

977 981

2 y x 3

$\begin{cases} x^2-y^2=31328 \\ x-y\equiv 8[22] \end{cases}$ y x

c a b a c b a (4

c b a

n

PGCD $PGCD(a;b^n)=1$

1954^{1962} 1962^{1954}

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1 2014 48

y x $2013x-1962y=54:(E)$ 1

(E) PGCD(2013;1962)

$x\equiv 0[6]$ (E) $(x;y)$

(E) 74 x_0 80 $(x_0;y_0)$

(E) $(x;y)$ y x d 2

d

$PGCD(a;b)=18$ $671a-654b=18$ b a

1 2013 49

$\beta=n+3$ $\alpha=2n^3-14n+2$ β α n .1

$PGCD(\alpha;\beta)=PGCD(\beta;10)$

$PGCD(\alpha;\beta)$

$PGCD(\alpha;\beta)=5$ n

11 4^n **2**

$$\begin{cases}4^{5n}+4^n+n\equiv 0[11]\\ n\equiv 2[10]\end{cases}$$ n

3 2013 50

$2n+27\equiv 0[n+1]$ n **1**

$(b-a)(a+b)=24$ $(a;b)$

$\sqrt{24}$

β α **2**

$\beta=\overline{3403}$ $\alpha=\overline{10141}$

β α

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$$\begin{cases} b^2 - a^2 = 24 \\ \alpha a - \beta b = 9 \end{cases}$$ ² $(a;b)$

1434 2013 3

478 671

$2013x - 1434y = 27$ $(x;y)$ ²

1 2012 51

$2011x - 1432y = 31....(1)$ $(x;y)$ ²

2011 1

1 1 $(x_0;y_0)$

7 2^n n 2

7 $2011^{1432^{2012}}$

$2010^n + 2011^n + 1432^n \equiv 0[7]$ n

γ β α 9 $\overline{2\gamma\alpha\beta}$ N 3

1 $(\beta;\gamma)$

N γ β α

1 2011 52

(E) y x $13x - 7y = -1...(E)$ 1

$$\begin{cases} a \equiv -1[7] \\ a \equiv 0[13] \end{cases}$$ a 2

13 7 9^n n 3

$\overline{\alpha 00 \beta 086}$ 9 b 4

$\alpha \neq 0$ β α

91 b β α

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1 2010 53

13 $3^{3n}-1$ $n\in$ 1

n 2

13 $3^{3n+2}-9$ $3^{3n+1}-3$

13 2005^{2010} 13 3^n n 3

$A_p=3^p+3^{2p}+3^{3p}$ p 4

13 A_p $p=3n$

13 A_p $p=3n+1$

$p=3n+2$ 13 A_p

3 b a 5

$b=\overline{1000100010000}$ $a=\overline{1001001000}$

A_p b a

13 b a

2 2010 54

$7x+65y=2009....(1)$ ² 1

7 y 1 $(x;y)$

1

9 2^n n 2

9 $2^{6n}+3n+2$ n 3

$u_n=2^{6n}-1$ n 4

9 u_n

y x $(x;y)$ $(7u_1)x+(u_2)y=126567....(2)$

$y_0\geq 25$ y_0 x_0 2 $(x_0;y_0)$

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1 2009 55

y 1 x

$A = \overline{5566}$ x A

y x $(5x^2+6)(x+1)$ 1

$A = (5x^2+6)(2+2y)$

12 x y x

A

584 2

$\begin{cases} a+b=32 \\ a^2+b^2=584 \end{cases}$ a b b a

2 2008 56

$3x-21y=78....(E)$ y x

² (E) 1

$x \equiv 5[7]$ (E) ² $(x;y)$

(E)

7 5^n n 2

$5^x+5^y \equiv 3[7]$ (E) ² $(x;y)$

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1997 57

2905 32785 2490 1

72 7 79 $7x+6y=79$ ² 2

2905 3

32785 2490

2001 58

170 993 1

$993x-170y=143....(1)$ y x 1 ² 1

$x_0+y_0=6$ 1 $(x_0;y_0)$

1 ²

340 1986 $(A-1)$ A 3

300 14

1998 59

$3x-5\equiv 0[11]$ x 1

$3x-11y=5....(1)$ ² 2

.

y x d 3

1 $(x;y)$ d

$d=5$ 1 ² $(x;y)$ 4

2008 60

$13x-11y=23....(1)$ ²

1 $x_0-y_0=1$ 1 $(x_0;y_0)$ 1

-10 x 40 1 $(x;y)$

d d y x 2

d 23 1 $(x;y)$

x $(x;y)$

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1995 61

1995

$PGCD(x;y)=19$ $x+7y=1995$ ² $(x;y)$

1996 62

$1\le a\le b\le c$ c b a

$bc=\overline{555}$ $b+c=\overline{46}$ a abc c b a

1997 63

2994 1497 1996 1

y x $1996x-1497y=3994....(1)$ 2

1 2 y 3 x

$x.y=1950$ 1 $(x;y)$

1992 64

580 1885 1

$1885x-580y=\alpha..(1)$ α 2

² 1 α

y x $(x;y)$ 1 $\alpha=1305$ 3

1992 65

$18x+4y=84$ ² 1

x.y 0 $(x;y)$

7 $\overline{55\alpha\beta}$ 5 $\overline{30\alpha\beta\gamma}$ n 2

n γ β α

1990 66

$5x-6y=3....(1)$ ²

(1) 3 x 1 $(x;y)$ 1

$\begin{cases} x\equiv -1[6] \\ x\equiv -4[5] \end{cases}$ 1 2

1 ² $(x;y)$ 3

(x^2-y^2) 56 $(x;y)$

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1996 67

xy $(x+y)$ y x 1

$15\alpha^2 - 229\beta = 30\beta$ β α β α 2

y x 3

$15(x^2+y^2)$ $229(x+y)$ $(x;y)$

1997 68

$(3,1)$ $9x'-14y'=13$ $(x';y')$ ² 1

$45x-28y=130$ ² 2

5 y 2 x $(x;y)$

7 $\overline{5\beta\beta6}$ 9 $\overline{2\alpha\alpha3}$ N 3

N β α

2005 69

$n \in$ $\beta = n+2$ $\alpha = n^2+n$ β α

$PGCD(\alpha;\beta) = PGCD(\beta;n)$ 1

$PGCD(\alpha;\beta)$

b $\overline{384}$ a $\overline{3520}$ n b a 2

b a $3n$ 2

$PGCD(a;b) = 2(3n+2)$ $PGCD(a;b) = 3n+2$ n

$PGCD(a;b) = 41$ β α

2007 70

c $2n$ 3 b $4n$ 3 a $2n$ 1 n 2 n

c b a b a 1

c b n 2

PPCM b,c 1305 PGCD b,c 3 n

a b^2 3

1994 71

10 2^n n 1

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1994 1414

u_n 2^n $u_{n\ n}$ 2

$u_{n\ n}$

S_n 5 2^1 5 2^2 ..... 5 2^n n

10 S_n n

2004 72

10 7^n n 1

10 $\left(7^{4k}+7^{4k+1}+7^{4k+2}+7^{4k+3}\right)$ k

$S_n = 1+7+7^2+...+7^n$ 2

$S_{n+4} \equiv S_n\,[10]$ n

10 S_n n

1980 73

1980 1

m^2 $5d^2$ 1980 b a 2

d $PGCD\ a,b$ m $PPCM\ a,b$

2007 74

y x $4862x - 1430y = 2002.....(E)$

2002 1430 4862 1

2 E 2

E

d d $PGCD\ a,b$ E (a,b) b a 3

d

$d = 7$ (a,b)

2009 75

5 4^{2n} 1 n

5 3^n 2

5 1429^{2009} 3

$A_n = 2+4^{2n}+3^n$ 4

5 A_n n

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2003 76

n 3 $3n^2$ 9n 16 n 1

n 3 $3n^3$ 11n 48 n

$3n^2$ 9n 16 n

c b a 2

PGCD a;b PGCD bc a;b

2 3

PGCD $3n^3$ 11n;n 3 PGCD 48;n 3

48 4

A $\frac{3n^3 \quad 11n}{n \quad 3}$ n

2005 77

y x $109x-226y=1$ (E) 1

(E) 226 109

(E)

k $(141+226k\,;68+109k)$

226 d

e d $109d=1+226e$ e

227 2

A g f $a\le 226$ a A **3**

227 a^{141} a g 227 a^{109} a f

$g\left[f(0)\right]=0$

1 $a^{226}\equiv 1[227]$ $a\in A-\{0\}$

$g\left[f(a)\right]=a$ $g\left[f(a)\right]=a$ $a\in A-\{0\}$

1981 78

b 13n 1 a 11n 3 b a a

50 b a 1

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50x 11y 3 2 2

b a 50 n 3

b a 25 n

2008 79

(E) 3x 8y 5.....(E) 2 1

k $(8k-1;3k-1)$

(E) (x,y) $\begin{cases} n=3x+2 \\ n=8y+7 \end{cases}$ y x n 2

n $\begin{cases} n\equiv 2[3] \\ n\equiv 7[8] \end{cases}$ (S)

$n\equiv 23[24]$ (S) n

8 7^{2k} 3 2^{2k} k 3

24 $1991^{2008}-1$ (S) 1991

2008 80

11x 5y 2 y x (E) (1

(E) (2;4)

11(x 2) 5(y 4) (E) (x;y)

(E)

$b=7n+5$ $a=5n+2$ n 2

PGCD a;b 11 PGCD a;b 1 5b – 7a

PGCD a;b 11 n 1

2011 81

$N=111.....11$

1م 2010 مرة الرقم

11 N 1

$10^{2010}-1=9N$ 2011 2

$9N$ 2011

N 2011

22121 N 3

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2008 82

12 $0,1,2,3,4,5,6,7,8,9,\alpha,\beta$

$N_1=\overline{\beta 1\alpha}^{12}$ 12 N_1 1

N_1

$N_2=1131$ N_2

12 N_2

$N=\overline{a_n.....a_1a_0}^{12}$ N 2

12 3 $N\equiv a_0[3]$ 3

12 N_2 12

11 $N\equiv a_n+...+a_1+a_0[11]$ 4

12

11 N_1 12

$\overline{x\,4y}^{12}$ 12 N 4

33 N y x

Antilles Guyane 2001 83

p $PGCD(a+b;ab)=p$ b a 1

$a^2=a(a+b)-ab$ a^2 p

b p a p

$PGCD(a;b)=p$

$a\le b$ b a 2

$\begin{cases}\text{PGCD}(a+b;ab)=5\\ \text{PPCM}(a;b)=170\end{cases}$ $\begin{cases}\text{PGCD}(a;b)=5\\ \text{PPCM}(a;b)=170\end{cases}$

Antilles Guyane 2011 84

y x $11x-7y=5$ (E) I

$11u-7v=5$ $(u;v)$

(E) (E) $(u;v)$

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(D) (O,i,j)

$M(x;y)$ $\mathscr{E}$ $11x-7y-5=0$

$0\le y\le 50$ $0\le x\le 50$

$\mathscr{E}$ (D)

y x $11x^2-7y^2=5$ (F) *II*

$x^2\equiv 2y^2[5]$ (F) $(x;y)$

y x

$y\equiv$	0	1	2	3	4	[5]
$2y^2\equiv$						[5]

$x\equiv$	0	1	2	3	4	[5]
$x^2\equiv$						[5]

5 $2y^2$ x^2

5 y x (F) $(x;y)$

(F) $(x;y)$ 5 y x *III*

(F)

Polynésie 2011 85

a p « *Fermat* »

a^{p-1} 1 p p

$u_{n+1}=10u_n+21$ $n\in$ $u_0=1$ (u_n)

u_3 u_2 u_1 1

$3u_n=10^{n+1}-7$ n 2

u_n n

u_2 3

u_n

5 3 2 u_n n 4

$3u_n\equiv 4-(-1)^n[11]$ n 5

11 u_n n

$10^{16}\equiv 1[17]$ 6

17 u_{16k+8} k

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2008 86

251 1

2008 2

2008

$m^3 + 35d^3 = 2008$ b a

$d = PGCD(a;b)$ $m = PPCM(a;b)$

87

7 5^n n 1

$4S_n = 5^{n+1} - 1$ $S_n = 1 + 5 + 5^2 + ... + 5^n$ $n \in$ 2

$S_n \equiv 2a[7]$ $4S_n \equiv a[7]$ a

7 S_{2016}

$(E): 5^n x - S_n y = 7$ $(E_0): 5^n x - S_n y = 0$ × 3

(E_o) 5^n S_n $n \in$

$k \in$ $y = 28 + k5^n$ $x = 35 + kS_n$ $(x;y)$ (E)

$\begin{cases} 25x - 31y = 7 \\ PGCD(x,y) = 7 \end{cases}$ ×

88

11 4^n n 1

11 $2015^{10n+4} - 10 \times 1434^{31}$

$x^2 + 2x + 9 \equiv 0[11]$ x 2

11 $5 \times 9^{4n+1} + 2^{2n+1} - 2$ n

5 $17^{4n+1} + 3 \times 9^{2n}$ n 3

$187 \times 17^{4n} + 11 \times 3^{4n+1} + 60 \times 4^{5n-1}$ 55

89

$b = 7n + 1$ $a = 8n + 1$ b a n

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b a 1

y x (E) $23x-26y=1$ 2

(E) $(a;b)$ n

(E) (E)

$\begin{cases} 0 \le a \le 25 \\ 23a \equiv 1[26] \end{cases}$ a 3

90

$u_{n+1}=10u_n+81$ n $u_0=1$ (u_n)

$u_{n+1}=10u_n+81$ n $u_0=1$

u_3 u_2 u_1 **1**

$u_n=10^{n+1}-9$ n **2**

$u_n+8=9+9\times10+9\times10^2+...+9\times10^n$ n

u_n n

u_2 **3**

5 3 2 u_n

7 u_n n **4**

$u_n \equiv 2-(-1)^n[11]$ n **5**

11 u_n n

91

(E) $3x-2y=1$ 2

(E) $(14n+3;21n+4)$ n

$21n+4$ $14n+3$

$21n+4$ $2n+1$ d

$d=13$ $n \equiv 6[13]$ $d=13$ $d=1$

$n \ge 2$ n

$A=21n^2-17n-4$ $B=28n^3-8n^2-17n-3$

$(n-1)$ B A

B A n

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92

580 1885 1

1885x 580y 1 2

1

1305 3

1

y x x, y

93

(E).....32x 7y 185 ^{2}

$4x$ 3 7 (E) $(x;y)$ (1

(E) ^{2} (2

7 5 9 2 8 n (3

n

2017 4

$(a^2$ 2017) a

94

$PPCM(a;b)$ 2160 $PGCD(a;b)$ 48 ^{2} $(a;b)$ 1

$9x$ 17 5 x 2

y x $432x$ $240y$ 816 3

3 5 2 9 n 4

17 45 n

2025 1980 n

2016 2016 5

$\overline{1202}x$ 2018

95

(E) $3x-2y=1$ ^{2}

(E) $(14n+3;21n+4)$ n

$21n+4$ $14n+3$

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$21n+4$ $2n+1$ d

$d=13$ $n\equiv 6[13]$ $d=13$ $d=1$

$A=21n^2-17n-4$ $B=28n^3-8n^2-17n-3$ $n\geq 2$ $n\in$

$(n-1)$ B A

B A n

96

5 3^n n 1

5 $(1962)^{1954}$ $(1439)^{2018}$

n 2

5 $(2018)^{4n\ 3}$ 2 $(1962)^{1954}$ $(1439)^{2018}$

5 $3^{4n\ 1}$ 2017^n 6 n 3

97

3^n n 1

$33^{16n+2}-2\times 109^{8n+1}-11\equiv 0[10]$ n 2

$10<n\leq 25$ $7\times 3^{n+1}-1\equiv 0[10]$ n 3

7 $\overline{y612}$ 3 $\overline{xx0xx02}$ A 4

A y x

A

98

(S) N 5 13 : N

(S) 239

y x N 1 $17x$ 5 $13y$ N

$17x$ $13y$ 4

y x $17x$ $13y$ 4 Z^2

(S) N 18 $221k$ k

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(E) $(6n+2;10n+3)$ **-1**

$5(6n+2)-3(10n+3)=1$. (E) $(6n+2;10n+3)$

$6n+2$ $10n+3$

$6n+2$ $10n+3$ $5(6n+2)-3(10n+3)=1$

$d=41$ $d=1$ **2**

$b=3n+5$ $a=10n+3$ $PGCD(a;b)=d$

n b a

$3a-10b=41$ $3a-10b=10(3n+5)-3(10n+3)=41$

1 3a d a d

2 10b d b d

41 d $3a-10b$ d 2 1

41 1 41

$n\equiv 12[41]$ $d=41$

$\begin{cases}10n+3\equiv 0[41]\\3n+5\equiv 0[41]\end{cases}$ $\begin{cases}a\equiv 0[41]\\b\equiv 0[41]\end{cases}$ $d=41$

6 $42n\equiv 12[41]$ $7n\equiv 2[41]$ $7n-2\equiv 0[41]$

$42\equiv 1[41]$ $n\equiv 12[41]$

$2n+3$ B A **3**

$B=6n^2+19n+15$ $A=20n^2+36n+9$

2n+3 A $A=20n^2+36n+9=(2n+3)(10n+3)$

2n+3 B $B=6n^2+19n+15=(2n+3)(3n+5)$

B A **n** **n**

$PGCD(A;B)=(2n+3)PGCD(a;b)$

$d=41$ $d=1$

$n\equiv 12[41]$ $PGCD(A;B)=41(2n+3)$ $d=41$

$k\in$ $n\neq 41k+12$ $PGCD(A;B)=(2n+3)$ $d=1$

2017 27

$4^{5k} \equiv 1[11]$ **k** **1**

$4^5 \equiv 1[11]$ $4^5 = 1024 = 11.93 + 1$

$4^{5k} \equiv 1[11]$ k

11 4^n **2**

11 4^n

5 11 4^n

n	5k	5k+1	5k+2	5k+3	5k+4
$4^n \equiv$	1	4	5	9	3

11 $2\times 2017^{5n+3} + 3\times 1438^{10n} + 1$ **n** **3**

$2017^{5n+3} \equiv 4^{5n+3}[11] \equiv 9[11]$ $1438^{10n} \equiv (8)^{10n}[11] \equiv 1[11]$

$2\times 2017^{5n+3} + 3\times 1438^{10n} + 1 \equiv 2\times 9 + 3\times 1 + 1[11]$

$2\times 2017^{5n+3} + 3\times 1438^{10n} + 1 \equiv 0[11]$ $2\times 2017^{5n+3} + 3\times 1438^{10n} + 1 \equiv 22[11]$

11 $2\times 2017^{5n+2} + n - 3$ **n** **4**

$2\times 2017^{5n+2} + n - 3 \equiv 0[11]$ 11 $2\times 2017^{5n+2} + n - 3$

$2017^{5n+2} \equiv 4^{5n+2}[11] \equiv 5[11]$

$p \in$ $n = 11p + 4$ $n + 7 \equiv 0[11]$ $2\times 2017^{5n+2} + n - 3 \equiv 0[11]$

2017 28

5 3^n n **1**

$3^4 \equiv 1[5]$ $3^3 \equiv 2[5]$ $3^2 \equiv 4[5]$ $3^1 \equiv 3[5]$ $3^0 \equiv 1[5]$

4 5 3^n

n =	4k	4k+1	4k+2	4k+3
$3^n \equiv$	1	3	4	2

5 1437^{2017} **2**

$1437^{2017} \equiv (-3)^{2017}[5]$ $1437 \equiv (-3)[5]$ $1437 \equiv 2[5]$

$1437^{2017} \equiv (-3)[5]$ $(-3)^{2017} \equiv (-1)^{2017}.3^{4(504)+1}[5]$

2017 $(-1)^{2017} = -1$ $3^{4(504)+1} \equiv 3[5]$

$(-3) \equiv 2[5]$ $1437^{2017} \equiv 2[5]$ $1437^{2017} \equiv (-3)[5]$

2 5 1437^{2017}

5 $(48^{4n+3}-2\times 9^{2n+1}+1)$ **n** **3**

$(48^{4n+3}-2\times 9^{2n+1}+1)\equiv 0[5]$ 5 $(48^{4n+3}-2\times 9^{2n+1}+1)$

$48^{4n+3}\equiv 2[5]$ $48^{4n+3}\equiv 3^{4n+3}[5]$ $48\equiv 3[5]$

$9^{2n+1}=3^{4n+2}\equiv 4[5]$

$(48^{4n+3}-2\times 9^{2n+1}+1)\equiv -5[5]$ $(48^{4n+3}-2\times 9^{2n+1}+1)\equiv 2-2\times 4+1[5]$

$(-5)\equiv 0[5]$ $(48^{4n+3}-2\times 9^{2n+1}+1)\equiv 0[5]$

5 $(3^{4n}+27^{n}-4)$ n 4

$(3^{4n}+27^{n}-4)\equiv 0[5]$ 5 $(3^{4n}+27^{n}-4)$

$3^{4n}\equiv 1[5]$ $27\equiv (3)^{3n}[5]$

$(3^{3n})\equiv 3[5]$ $(3^{4n}+27^{n}-4)\equiv 0[5]$

$p\in$ $n=4p+3$ $n\equiv 3[4]$ $3n\equiv 1[4]$

1 2016 29

(E) (E) $(x_0;y_0)$ 1

$x_0=y_0=-19$ $6x_0-7x_0=-19$ $x_0=y_0$ (E) $(x_0;y_0)$

$$\begin{cases}6x-7y=19\\6x_0-7x_0=19\end{cases}$$

$6(x-x_0)=7(y-y_0).....(1)$ $6(x-x_0)-7(y-y_0)=0$

7 6 $(y-y_0)$ 6 $7(y-y_0)$ 6 1

$y=6k-19$ $(y-y_0)=6k$

$x=6k-19$ 1 y

k $(x;y)=(7k-19;6k-19)$ (E)

42 λ $\begin{cases}\lambda\equiv 24[7]\\\lambda\equiv 5[6]\end{cases}$ λ **2**

$6\beta-7\alpha=19...(E')$ $\begin{cases}\lambda=7\alpha+24\\\lambda\equiv 6\beta+5\end{cases}$ $\begin{cases}\lambda\equiv 24[7]\\\lambda\equiv 5[6]\end{cases}$

$\beta=x=7k-19$ (E') (E)

$\lambda\equiv 42k-119$ $\lambda\equiv 6(7k-19)+5$

$-119\equiv 7[42]$ $\lambda\equiv 42k+7$ $\lambda\equiv 42k-119$

7 42 λ

$|x+y-1|\le 13$ (E) $(x;y)$ **3**

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$|x+y-1| \le 13 \quad (x;y) = (7k-19;6k-19)$

$2 \le k \le 4 \quad |k-3| \le 1 \quad |13k-39| \le 13 \quad |7k+6k-39| \le 13$

$(9;5) \quad (2;-1) \quad (-5;-7) \quad (x;y) \quad k \in \{2;3;4\}$

4 $7 \quad 5^n$

$5^6 \equiv 1[7] \quad 5^5 \equiv 3[7] \quad 5^4 \equiv 2[7] \quad 5^3 \equiv 6[7] \quad 5^2 \equiv 4[7] \quad 5^1 \equiv 5[7] \quad 5^0 \equiv 1[7]$

$6 \qquad 7 \qquad 5^n$

$n =$	$6k$	$6k+1$	$6k+2$	$6k+3$	$6k+4$	$6k+5$
$5^n \equiv$	1	5	4	6	2	3

$\begin{cases} n - 5^n \equiv 2020[7] \\ n \equiv 1437[6] \end{cases}$ **n**

$\begin{cases} 6k+3 \equiv 5^{6k+3} + 4[7] \\ n \equiv 6k+3 \end{cases} \quad \begin{cases} n - 5^n \equiv 4[7] \\ n \equiv 3[6] \end{cases} \quad \begin{cases} n - 5^n \equiv 2020[7] \\ n \equiv 1437[6] \end{cases}$

$p \qquad n = 42p + 3 \quad \begin{cases} k \equiv 7p \\ n \equiv 6k+3 \end{cases} \quad \begin{cases} k \equiv 0[7] \\ n \equiv 6k+3 \end{cases}$

1 2015 30

13 8^n (**1**

$8^4 \equiv 1[13] \quad 8^3 \equiv 5[13] \quad 8^2 \equiv 12[13] \quad 8^1 \equiv 8[13] \quad 8^0 \equiv 1[13]$

$4 \qquad 13 \qquad 8^n$

$n =$	$4k$	$4k+1$	$4k+2$	$4k+3$
$8^n \equiv$	1	8	12	5

13 $42 \times 138^{2015} + 2014^{2037} - 3$

$138^{2015} \equiv 8^{2015}[13] \quad 138 \equiv 8[13] \quad 42 \equiv 3[13]$

$138^{2015} \equiv 5[13] \quad 8^{2015} \equiv 5[13] \quad 2015 \equiv 3[4]$

$12 \equiv -1[13] \quad 2014 \equiv -1[13] \quad 2014 \equiv 12[13]$

$2037 \quad 2014^{2037} \equiv (-1)[13] \quad 2014^{2037} \equiv (-1)^{2037}[13]$

$42 \times 138^{2015} + 2014^{2037} - 3 \equiv 3 \times 5 - 1 - 3[13]$

$42 \times 138^{2015} + 2014^{2037} - 3 \equiv 11[13]$

$11 \quad 13 \quad 42 \times 138^{2015} + 2014^{2037} - 3$

$(5n+1) \times 64^n - 5^{2n+3} \equiv (5n+6) \times 8^{2n}[13] \quad n \in$ (**2**

$5^{2n+3} \equiv (-8)^{2n+3}[13] \quad 5 \equiv -8[13] \quad 64^n = 8^{2n}$

$(-8)^{2n+3} \equiv -5.(8)^{2n}\,[13]$ $(-8)^{2n+3} \equiv (8)^{2n}(-1)^{2n+3}.8^3\,[13]$

$(5n+1)\times 64^n - 5^{2n+3} \equiv (5n+1)\times 8^{2n} + 5\times 8^{2n}\,[13]$

$(5n+1)\times 64^n - 5^{2n+3} \equiv (5n+6)\times 8^{2n}\,[13]$

$(5n+1)\times 64^n - 5^{2n+3} \equiv 0[13]$ **n**

$(5n+1)\times 64^n - 5^{2n+3} \equiv (5n+6)\times 8^{2n}\,[13]$

$(5n+6)\times 8^{2n} \equiv 0[13]$ $(5n+1)\times 64^n - 5^{2n+3} \equiv 0[13]$

13 8^{2n} 13 8 $(5n+6) \equiv 0[13]$

$-6 \equiv 20[13]$ $5n \equiv 20[13]$ $5n \equiv -6[13]$ $(5n+6) \equiv 0[13]$

p $n = 13p+4$ $n \equiv 4[13]$ $5n \equiv 20[13]$

1 2013 31

$x_0 + y_0 = -1$ (E) $(x_0; y_0)$ 1

$\begin{cases} 11x_0 + 7y_0 = 1 \\ -7x_0 - 7y_0 = 7 \end{cases}$ $\begin{cases} 11x_0 + 7y_0 = 1 \\ x_0 + y_0 = -1 \end{cases}$ $(x_0; y_0)$

$y_0 = -1 - x_0 = -3$ $x_0 = 2$ $4x_0 = 8$

$(x_0; y_0) = (2; -3)$

(E)

(E) $(x_0; y_0) = (2; -3)$

$11(x-2) + 7(y+3) = 0$ $\begin{cases} 11x + 7y = 1 \\ 11(2) + 7(-3) = 1 \end{cases}$

$11(x-2) = 7(-y-3).....(*)$ $11(x-2) + 7(y+3) = 0$

11 7 $7/(x-2)$ $7/11(x-2)$ (*)

k $x = 7k+2$ $x - 2 = 7k$

$y = -11k - 3$ $11(7k) = 7(-y-3)$ (*) x

$(x; y) = (7k+2; -11k-3)$ (E)

(E) $(a; -b)$ 2

$11a + 7(-b) = 1$ $11a + 1 = 7b + 2$ $\begin{cases} S = 11a + 1 \\ S = 7b + 2 \end{cases}$

(E) $(a; -b)$ $11a + 7(-b) = 1$

77 S

$(a;-b)=(7k+2;-11k-3)$ (E) $(a;-b)$

$S=11(7k+2)+1=77k+23$ $S=11a+1$ a $a=7k+2$

77 S 23 $S=77k+23$

1 2012 32

11 9^n n 1

$9^5\equiv 1[11]$ $9^4\equiv 5[11]$ $9^3\equiv 3[11]$ $9^2\equiv 4[11]$ $9^1\equiv 9[11]$ $9^0\equiv 1[11]$

5 11 9^n

n =	5k	5k+1	5k+2	5k+3	5k+4	
$9^n\equiv$	1	9	4	3	5	[11]

11 2011^{2012} 2

$2011^{2012}\equiv 9^{2012}[11]$ $2011\equiv 9[11]$

$9^{2012}=9^{5k+2}\equiv 4[11]$ $2011^{2012}\equiv 4[11]$ $2012\equiv 2[5]$

4 11 2011^{2012}

11 $4\times 9^{15n+1}+4\times 2011^{10n}+2011^{2012}$ n 3

$(9^{5n})\equiv 1[11]$ $9^{15n+1}=(9^{5n})^3\times 9\equiv 9[11]$

$(9^{5n})\equiv 1[11]$ $2011^{10n}\equiv 1[11]$ $2011^{10n}\equiv (9^{5n})^2[11]$

$2011^{2012}\equiv 4[11]$

$4\times 9^{15n+1}+4\times 2011^{10n}+2011^{2012}\equiv 4\times 9+4\times 1+4[11]$

$4\times 9+4\times 1+4=44=4\times 11$ $4\times 9^{15n+1}+4\times 2011^{10n}+2011^{2012}\equiv 0[11]$

11 $2011^{2012}+2n+2$ n 4

$2011^{2012}+2n+2\equiv 0[11]$ 11 $2011^{2012}+2n+2$

$2011^{2012}\equiv 4[11]$ $4+2n+2\equiv 0[11]$ $2011^{2012}+2n+2\equiv 0[11]$

$n\equiv -3[11]$ $2n\equiv -6[11]$

$\alpha\in$ $n=11\alpha+8$ $n\equiv 8[11]$

2 2012 33

(S) 153 1

(S) 153 $\begin{cases}153\equiv 3[15]\\153\equiv 6[7]\end{cases}$ $\begin{cases}153=15\times 10+3\\153=7\times 21+6\end{cases}$

$\begin{cases}x-x_0\equiv 0[15]\\x-x_0\equiv 0[7]\end{cases}$ (S) x 2

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$\left(\begin{cases} x - x_0 \equiv 0[15] \\ x - x_0 \equiv 0[7] \end{cases}\right)$ (S) x (S) x_0 1

$\begin{cases} x \equiv 3[15] \\ x \equiv 6[7] \end{cases}$....(2) (S) x $\begin{cases} x_0 \equiv 3[15] \\ x_0 \equiv 6[7] \end{cases}$....(1)

$\left(\begin{cases} x - x_0 \equiv 0[15] \\ x - x_0 \equiv 0[7] \end{cases}\right)$ $\begin{cases} x \equiv x_0[15] \\ x \equiv x_0[7] \end{cases}$ 2 1

(S) x (S) x_0 $\left(\begin{cases} x - x_0 \equiv 0[15] \\ x - x_0 \equiv 0[7] \end{cases}\right)$ 2

$\begin{cases} x_0 \equiv 3[15] \\ x_0 \equiv 6[7] \end{cases}$....(1) $\left(\begin{cases} x - x_0 \equiv 0[15] \\ x - x_0 \equiv 0[7] \end{cases}\right)$....(2)

(S) x $\begin{cases} x \equiv 3[15] \\ x \equiv 6[7] \end{cases}$ 2 1

(S) 3

1 $x \equiv 153[105]$ $\begin{cases} 7x \equiv 21[105].....(1) \\ 15x \equiv 90[105].....(2) \end{cases}$ $\begin{cases} x \equiv 3[15] \\ x \equiv 6[7] \end{cases}$

k $x = 105k + 48$ $153 \equiv 48[105]$ $x \equiv 48[105]$

4

x

$x = 15\alpha + 3$ 3 15

$x = 7\beta + 6$ 6 7

500 x 600 (S) x

500 $105k + 48$ 600 $x = 105k + 48$ 3

$k = 5$ $\frac{452}{105}$ k $\frac{552}{105}$ 452 105k 552 500 $105k + 48$ 600

$x = 573$ $x = 105k + 48$ k

2 2011 34

$A_3 \equiv 6[7]$ $4 \equiv -3[7]$ 1

$4 \equiv -3[7]$ $4 + 3 \equiv 0[7]$ $7 \equiv 0[7]$

$A_3 = 2^3 + 3^3 + 4^3 + 5^3 + 6^3$ $A_n = 2^n + 3^n + 4^n + 5^n + 6^n$

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$$6\equiv -1[7] \quad 5\equiv -2[7] \quad 4\equiv -3[7]$$

$$A_3\equiv 2^3+3^3+(-3)^3+(-2)^3+(-1)^3[7]$$

$$A_3\equiv 2^3+3^3-(3)^3-(2)^3-(1)^3[7]$$

$$-1\equiv 6[7] \qquad A_3\equiv 6[7] \qquad A_3\equiv -1[7]$$

7 3^n 2^n n 2

$7 \quad 2^n$

$$2^3\equiv 1[7] \quad 2^2\equiv 4[7] \quad 2^1\equiv 2[7] \quad 2^0\equiv 1[7]$$

3 7 2^n

$n=$	$3k$	$3k+1$	$3k+2$
$2^n\equiv$	1	2	4

$7 \quad 3^n$

$$3^6\equiv 1[7] \quad 3^5\equiv 5[7] \quad 3^4\equiv 4[7] \quad 3^3\equiv 6[7] \quad 3^2\equiv 2[7] \quad 3^1\equiv 3[7] \quad 3^0\equiv 1[7]$$

6 11 3^n

$n=$	$6k'$	$6k'+1$	$6k'+2$	$6k'+3$	$6k'+4$	$6k'+5$
$3^n\equiv$	1	3	2	6	4	5

7 A_n+1 n 3

$n=2p+1$ n $A_n=2^n+3^n+4^n+5^n+6^n$

$$(-a)^{2p+1}=-a^{2p+1}$$

$A_n+1\equiv 0[7]$ $A_n=2^{2p+1}+3^{2p+1}-3^{2p+1}-2^{2p+1}-1^{2p+1}=-1$

7 A_{2011}

6 $A_{2011}\equiv 6[7]$ $A_{2011}+1\equiv 0[7]$ 2011

7 A_{1432} 4

$$A_{1432}=2^{1432}+3^{1432}+4^{1432}+5^{1432}+6^{1432}$$

$A_{1432}=2^{1432}+3^{1432}+3^{1432}+2^{1432}+1^{1432}=2^{2864}+2.3^{1432}+1$ $(-a)^{2p}=a^{2p}$

$6k'+4$ $1432=6(238)+4$ $3k+2$ $2864=3(954)+2$

6 $A_{1432}\equiv 6[7]$ $A_{1432}=2^{2864}+2.3^{1432}+1\equiv 4+2.4+1[7]$

1 2010 35

3 n α 1

7 n

$0 \le \alpha \le 6$ α $n = \overline{11\alpha00}$

$n = \overline{11\alpha00} = 1.7^4 + 1.7^3 + \alpha.7^2 + 0.7 + 0.7^0 = 49\alpha + 2744$

$\alpha \equiv 0[3]$ 3 n α

$2744 \equiv 2[3]$ $49 \equiv 1[3]$ $\alpha + 2 \equiv 0[3]$ $49\alpha + 2744 \equiv 0[3]$ $\alpha \equiv 0[3]$

$0 \le \alpha \le 6$ $\alpha = 4$ $\alpha = 1$ $\alpha = 3k + 1$ $\alpha \equiv 1[3]$ $\alpha + 2 \equiv 0[3]$

5 n α 2

$\alpha \equiv 0[5]$ 5 n α

$2744 \equiv -1[5]$ $49 \equiv -1[5]$ $\alpha + 1 \equiv 0[5]$ $49\alpha + 2744 \equiv 0[5]$ $\alpha \equiv 0[5]$

$0 \le \alpha \le 6$ $\alpha = 4$ $\alpha = 5k + 4$ $\alpha \equiv 4[5]$ $\alpha + 1 \equiv 0[5]$

15 n α

3 5 n 15 n α

2 1 $\alpha = 4$

n $\alpha = 4$ 3

$n = 49(4) + 2744 = 2940$ $n = 49\alpha + 2744$

2 2010 36

13 10^n n 1

$10^6 \equiv 1[13]$ $10^5 \equiv 4[13]$ $10^4 \equiv 3[13]$ $10^3 \equiv 12[13]$ $10^2 \equiv 9[13]$ $10^1 \equiv 10[13]$ $10^0 \equiv 1[13]$

6 13 10^n

n =	6k'	6k'+1	6k'+2	6k'+3	6k'+4	6k'+5
$10^n \equiv$	1	10	9	12	3	4

$(10^{2008})^2 + 10^{2008} + 1 \equiv 0[13]$ 2

$10^{2008} \equiv 3[13]$ 6k'+4 $2008 = 6(334) + 4$

$(10^{2008})^2 + 10^{2008} + 1 \equiv 13[13]$ $(10^{2008})^2 + 10^{2008} + 1 \equiv 3^2 + 3 + 1[13]$

$(10^{2008})^2 + 10^{2008} + 1 \equiv 0[13]$

$10^{2n} + 10^n + 1 \equiv 0[13]$ n 3

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n

$n=$	$6k'$	$6k'+1$	$6k'+2$	$6k'+3$	$6k'+4$	$6k'+5$
$10^n \equiv$	1	10	9	12	3	4
$10^{2n} \equiv$	1	9	3	1	9	3
$10^{2n}+10^n+1 \equiv$	3	7	0	1	0	7

k' $n=6k'+4$ $n=6k'+2$

2 2009 37

$y'=(\ln 2)y$ 1

$y=c.e^{ax}-\frac{b}{a}$ $y'=ay+b$

c $y=c.e^{x\ln 2}=c.2^x$ $y'=(\ln 2)y$

$f(x)$ $f(0)=1$ 2

$f(x)=2^x$ $c=1$ $c.2^0=1$ $f(0)=1$ $y=f(x)=c.2^x$

2^n 7 3

$2^3\equiv 1[7]$ $2^2\equiv 4[7]$ $2^1\equiv 2[7]$ $2^0\equiv 1[7]$

3 7 2^n

$n=$	$3k$	$3k+1$	$3k+2$
$2^n \equiv$	1	2	4

$f(2009)-4$ 7

$3k+2$ $2009=3(669)+2$ $f(2009)=2^{2009}$

$2^{3k+2}\equiv 4[7]$ $f(2009)-4\equiv 0[7]$ $f(2009)-4\equiv 2^{3k+2}-4[7]$

0 $f(2009)-4$ 7

S_n n 4

$S_n=f(0)+f(1)+...f(n)=2^0+2+2^2+....2^n$

2 1 S_n

$S_n=1.\frac{2^{n+1}-1}{2-1}=2^{n+1}-1$

7 S_n n

$S_n\equiv 0[7]$ 7 S_n

$k\in$ * $n=3k-1$ $n+1=3k$ $2^{n+1}\equiv 1[7]$ $S_n\equiv 0[7]$

b a

$b-2a=7$ $d=PGCD(a;b)$

$d=7$ $d=1$ 7 d $b-2a$ d b a d

7 $n+5$ 7 b a

$\begin{cases}2n+3\equiv 0[7]....(1)\\ n-2\equiv 0[7].....(2)\end{cases}$ $\begin{cases}a\equiv 0[7]\\ b\equiv 0[7]\end{cases}$ 7 b a

7 $n+5$ $n+5\equiv 0[7]$ 1 2

$PGCD(a;b)=7$ n

$n+5\equiv 0[7]$ $\begin{cases}2n+3\equiv 0[7]....(1)\\ n-2\equiv 0[7].....(2)\end{cases}$ $PGCD(a;b)=7$

0 k $n=7k+2$ $n+5=7k$ $n+5\equiv 0[7]$

$n-5$ q p

$n-5$ p $p=2n^2-7n-15=(n-5)(2n+3)$

$n-5$ p $q=n^2-7n+10=(n-5)(n-2)$

$PGCD(p;q)$ n n

$q=n^2-7n+10=(n-5)a$ $p=2n^2-7n-15=(n-5)b$

$PGCD(p;q)=(n-5)PGCD(a;b)$

$PGCD(a;b)=7$ $PGCD(a;b)=1$

$n\neq 7k+2$ 5 n $PGCD(p;q)=(n-5)$ $PGCD(a;b)=1$ 1

$n=7k+2$ 5 n $PGCD(p;q)=7(n-5)$ $PGCD(a;b)=7$ 2

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1 (1) (82;1) 1

$4(82)-9(1)=319$ (1) (82;1)

$4(x-82)-9(y-1)=0$ 1 2 $\begin{cases}4x-9y=319...(1)\\4(82)-9(1)=319.....(2)\end{cases}$

$4(x-82)=9(y-1).....(*)$

4 9 $(x-82)$ 9 $4(x-82)$ 9 (*)

$y=4k+1$ (*) x $x=9k+82$ $(x-82)=9k$

k $(x;y)=(9k+82;4k+1)$ $(x;y)$

$4a^2-9b^2=319....(2)$ $(a;b)$ 2

$(2a-3b)(2a+3b)=319$ $4a^2-9b^2=319....(2)$

319

$D_{319}=\{1;11;29;319\}$ $(1+1)(1+1)=4$ $319=11\times 29$

$(-a;-b)$ 2 $(a;b)$ 1

$2a-3b$ $2a+3b$ 2

1 2 $\begin{cases}2a-3b=1...(1)\\2a+3b=319.....(2)\end{cases}$ $(2a-3b)(2a+3b)=1\times 319$ 1

$b=53$ 1 a $a=80$ $4a=320$

1 2 $\begin{cases}2a-3b=11...(1)\\2a+3b=29.....(2)\end{cases}$ $(2a-3b)(2a+3b)=11\times 29$ 2

$(a;b)$ $b=3$ 1 a $a=10$ $4a=40$

$(10;-3)$ $(-10;3)$ $(-10;-3)$ $(10;3)$ $(80;-53)$ $(-80;53)$ $(-80;-53)$ $(80;53)$

y_0 x_0 1 $(x_0;y_0)$

$y_0=b^2$ $x_0=a^2$ $4x_0-9y_0=319....(2)$ $4a^2-9b^2=319....(2)$

y_0 x_0 1 $(x_0;y_0)$

$(100;9)$ $(640;2809)$

2018 40

1 β $\frac{\alpha}{2}$ β α

$$\begin{cases}\alpha+\beta=4035\\ \alpha-\beta=1\end{cases}$$

β α

$$\begin{cases}\alpha=2018\\ \beta=\alpha-1=2017\end{cases} \qquad \begin{cases}2\alpha=4036\\ \alpha=1+\beta\end{cases} \qquad \begin{cases}\alpha+\beta=4035\\ \alpha-\beta=1\end{cases}$$

$(x_0;y_0)\in$ ² $\frac{\alpha}{2}x_0+\beta y_0=1$ β $\frac{\alpha}{2}$

β $\frac{\alpha}{2}$ $(x_0;y_0)=(2;-1)$ $1009x_0+2017y_0=1$

$$PGCD(\beta;\frac{\alpha}{2})=1$$

2 $(x;y)$ $1009x-2017y=1$.

$1009(2)-2017(1)=1$ $1009(2)+2017(-1)=1$

$1009x-2017y=1...(1)$ $(2;-1)$ $1009(2)-2017(1)=1$

$$\begin{cases}1009x-2017y=1\\ 1009(2)-2017(1)=1\end{cases}$$

$1009(x-2)=2017(y-1).....(1)$ $1009(x-2)-2017(y-1)=0$

1009 2017 $(y-1)$ 1009 $2017(y-1)$ 1009 1

$y=1009k+1$ $(y-1)=1009k$

$x=2017k+2$ 1 y

k $(x;y)=(2017k+2;1009k+1)$

3 **a**

$$\begin{cases}a\equiv 2019[2017]\\ a\equiv 2019[1009]\end{cases}$$

$2019\equiv 1[1009]$ $2019\equiv 2[2017]$ $\begin{cases}a\equiv 2[2017]\\ a\equiv 1[1009]\end{cases}$ $\begin{cases}a\equiv 2019[2017]\\ a\equiv 2019[1009]\end{cases}$

$1009\beta-2017\alpha=1...(2)$ $\begin{cases}a=2017\alpha+2\\ a=1009\beta+1\end{cases}$

$\beta=x=2017k+2$ $\alpha=y=1009k+1$ 2 1

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$k \in$ $a = 2017(1009k+1)+2 = 2035153k+2019$

9 7^n **4**

$7^3 \equiv 1[9]$ $7^2 \equiv 4[9]$ $7^1 \equiv 7[9]$ $7^0 \equiv 1[9]$

3 9 7^n

$n =$	$3k$	$3k+1$	$3k+2$
$7^n \equiv$	1	7	4

9 42L

$L = \overline{111...1}$ 7 L

$L = \overline{111...1} = 7^0 + 7^1 + 7^2 + ... + 7^{2017}$

$L = \dfrac{7^{2018}-1}{6}$ 7 1 L

42 $42L = 7^{2019} - 7$ $L = \dfrac{7^{2018}-1}{6}$

$7^{2019} \equiv 7[9]$ $7^{2019} - 7 \equiv 0[9]$

1 2017 41

(E) 20 104 **1**

20 104

$20 = 4 \times 5 + 0$ $104 = 20 \times 5 + 4$ 1

4 20 104

$20 = 2^2 \times 5$ $104 = 2^3 \times 13$ 2

$2^2 = 4$ 20 104

$104x - 20y = 272....(E)$

$272 = 4 \times 68$ 272 20 104 (E)

$x \equiv 3[5]$ (E) $(x;y)$

4 (E) $26x - 5y = 68....(E')$ (E)

$26x \equiv 68[5]$ $26x = 68 + 5y$ (E')

$68 \equiv 3[5]$ $26 \equiv 1[5]$ $x \equiv 3[5]$

(E)

(E') (E)

k $x = 5k+3$ $x \equiv 3[5]$

x $5y=-68+26x$ (E')

$y=21k+2$ $5y=-68+26(5k+3)=5(21k+2)$

k $(x;y)=(5k+3;26k+2)$ (E)

λ β α 2

4 $\overline{1\alpha\alpha\beta01}$ λ

$\lambda=1\times4^5+\alpha\times4^4+\alpha\times4^3+\beta\times4^2+1=1025+320\alpha+16\beta....(1)$

6 $\overline{1\alpha\beta01}$ λ

$\lambda=1\times6^4+\alpha\times6^3+\beta\times6^2+1=1267+216\alpha+36\beta....(2)$

$1025+320\alpha+16\beta=1297+216\alpha+36\beta$ 2 1

$0\le\beta\le3$ $0\le\alpha\le3$ $104\alpha-20\beta=272$

$104x-20y=272....(E)$ $104\alpha-20\beta=272$

$\beta=y=21k+2$ $\alpha=x=5k+3$

$(\alpha;\beta)=(3;2)$ $k=0$ $0\le\beta\le3$ $0\le\alpha\le3$

$\lambda=2017$ $(\alpha;\beta)=(3;2)$ $\lambda=1025+320\alpha+16\beta$

1009 2017 3

$\sqrt{1009}=31,76$ $\sqrt{2017}=44,91$

43 2017

31 1009

$2m-d=2017$ $(a;b)$

$m=PPCM(a;b)$ $d=PGCD(a;b)$

$PGCD(a';b')=1$ $b=b'.d$ $a=a'.d$ $d=PGCD(a;b)$

$m=d\times a'\times b'$ $m\times d=a\times b$

$2a'\times b'-1=\frac{2017}{d}...(*)$ $2m-d=2017$ $m=d\times a'\times b'$

2017 $d=2017$ $d=1$ 2017 d (*)

$a'\times b'=1009$ (*) $d=1$ 1

1009 $(a';b')=(1;1009)$ $(a';b')=(1009;1)$

$a'\times b'=1$ (*) $d=2017$ 2

b' a'

$(a;b)=(1;1009)$ $(a;b)=(1009;1)$ 1

$3u_n = 7^{n+1} - 4$ n 1

$u_{n+1} = 7u_n + 8 : n$ $u_0 = 1$ (u_n)

P(0)

$u_0 = 1$ $3u_0 = 3$ n=0

$3u_{n+1} = 7^{n+2} - 4$ P(n+1) $3u_n = 7^{n+1} - 4$ P(n)

$3u_{n+1} = 7(3u_n) + 24$ $u_{n+1} = 7u_n + 8 : n$

$3u_{n+1} = 7(7^{n+1} - 4) + 24$ $3u_n = 7^{n+1} - 4$

$3u_{n+1} = 7^{n+2} - 4$ $3u_{n+1} = 7.7^{n+1} - 28 + 24$

n $3u_n = 7^{n+1} - 4$

S_n n 2

7 1 $S_n = 1 + 7 + 7^2 ..... + 7^n$

$$S_n = 1\left(\frac{7^{n+1} - 1}{7 - 1}\right) = \frac{7^{n+1} - 1}{6}$$

S'_n S_n

$3u_n = 7^{n+1} - 4$ $S_n' = u_0 + u_1 + ... + u_n$

$3S_n' = 3u_0 + 3u_1 + ... + 3u_n$

$$3S_n' = \left[(7-4) + (7^2 - 4) + ... + (7^{n+1} - 4)\right] = 7(1 + 7 + 7^2 + ...7^n) - 4(n+1)$$

$3S_n' = 7S_n - 4(n+1)$

$18 \times S'_n = 7^{n+2} - 24n - 31$ n

6 $3S_n' = 7S_n - 4(n+1)$

$S_n = \frac{7^{n+1} - 1}{6}$ $18S_n' = 7(6S_n) - 24(n+1)$

$18 \times S'_n = 7^{n+2} - 24n - 31$ $18S_n' = 7(6\frac{7^{n+1} - 1}{6}) - 24(n+1)$

5 7^n n 3

$7^4 \equiv 1[5]$ $7^3 \equiv 3[5]$ $7^2 \equiv 4[5]$ $7^1 \equiv 2[5]$ $7^0 \equiv 1[5]$

4 5 7^n

n =	4k	4k + 1	4k + 2	4k + 3
$7^n \equiv$	1	2	4	3

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5 S'_n n

18 $18S'_n \equiv 0[5]$ $S'_n \equiv 0[5]$ 5 S'_n

$7^{n+2} + n - 1 \equiv 0[5]$ $7^{n+2} - 24n - 31 \equiv 0[5]$

$(k' \in \quad) n = 20k' + 12$ $k \equiv 3[5]$ $7^{n+2} + n - 1 \equiv 0[5]$ $n = 4k$ 1

$(k' \in \quad) n = 20k' + 13$ $k \equiv 3[5]$ $7^{n+2} + n - 1 \equiv 0[5]$ $n = 4k + 1$ 2

$(k' \in \quad) n = 20k' + 10$ $k \equiv 2[5]$ $7^{n+2} + n - 1 \equiv 0[5]$ $n = 4k + 2$ 3

$(k' \in \quad) n = 20k' + 19$ $k \equiv 4[5]$ $7^{n+2} + n - 1 \equiv 0[5]$ $n = 4k + 3$ 4

2017 43

63 5 **1**

$PGCD(63;5) = 1$ 63 5

$PGCD(63;5) = 1$

$3 = 2.1 + 1$ $5 = 3.1 + 2$ $63 = 5.12 + 3$

$PGCD(63;5) = 1$ 1

(E)

c $PGCD(a;b)$ ² $ax + by = c$

$63x + 5y = 159...(E)$

159 $PGCD(63;5) = 1$ ² (E)

$x \equiv 3[5]$ (E) $(x;y)$ **2**

$63x \equiv 159[5]$ $63x = 159 - 5y$ $63x + 5y = 159...(E)$

$159 \equiv 9[5]$ $63 \equiv 3[5]$ $3x \equiv 9[5]$

5 3 $x \equiv 3[5]$ $3x \equiv 9[5]$

(E)

k $x = 5k + 3$ $x \equiv 3[5]$

$5y = 159 - 63x$ $63x + 5y = 159...(E)$

$5y = 159 - 63x$ $x = 5k + 3$

$y = -63k - 6$ $-5y = 30 + 63 \times 5k$ $-5y + 159 = 63(5k + 3)$

$\{(x;y) = (5k + 3; -63k - 6); k \in \quad\}$ (E)

β α **3**

7 $\overline{5\alpha 0\alpha}$ λ

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$\lambda = \overline{5\alpha 0\alpha}^{7} = \alpha + 7^2\alpha + 7^3.5 = 50\alpha + 1715 .....(1)$

5 $\overline{\beta 10\beta 0}$ λ

$\lambda = \overline{\beta 10\beta 0}^{5} = 5\beta + 1.5^3 + 5^4\beta = 630\beta + 125 .....(2)$

$63\beta + 5(-\alpha) = 159....(3)$ $630\beta + 125 = 50\alpha + 1715$ 2 1

$(\beta;-\alpha) = (x;y) = (5k+3;-63k-6)$ (E) 3

$0 \le \beta \le 4$ $0 \le \alpha \le 6$ $(\beta;\alpha) = (5k+3;63k+6)$

$(\beta;\alpha) = (3;6)$ $k = 0$ $0 \le 63k+6 \le 4$ $0 \le 5k+3 \le 6$

$\lambda + 2$

$\alpha = 6$ $\lambda + 2 = 50\alpha + 1717$ $\lambda = 50\alpha + 1715$ 1

$\lambda + 2 = 50(6) + 1717 = 2017$

2

n **5** 3^n **4**

$3^4 \equiv 1[5]$ $3^3 \equiv 2[5]$ $3^2 \equiv 4[5]$ $3^1 \equiv 3[5]$ $3^0 \equiv 1[5]$

4 5 3^n

$n =$	$4k$	$4k+1$	$4k+2$	$4k+3$
$3^n \equiv$	1	3	4	2

5 $3^{x-y} + 4n + 1438^{2017}$ **n**

$3^{x-y} + 4n + 1438^{2017} \equiv 0[5]$ 5 $3^{x-y} + 4n + 1438^{2017}$

x $(x;y) = (5k+3;-63k-6)$

$3^{68k+8} \equiv 1[5]$ $3^{x-y} = 3^{68k+9} = 3.3^{68k+8} \equiv 3[5]$ $(x-y) = (68k+9)$

$3^{2017} \equiv 3[5]$ $1428^{2017} \equiv 3[5]$ $1428^{2017} \equiv 3^{2017}[5]$ $1428 \equiv 3[5]$

$3 + 4n + 3 \equiv 0[5]$ $3^{x-y} + 4n + 1438^{2017} \equiv 0[5]$

$p \in$ $n = 5p + 1$ $n \equiv 1[5]$ $4n \equiv 4[5]$ $3 + 4n + 3 \equiv 0[5]$

2017 44

$u_n = \frac{1}{3}(4^n - 1)$ n **1**

$u_{n+1} = 4u_n + 1$ $u_0 = 0$ (u_n)

n $u_n = \frac{1}{3}(4^n - 1)$

P(0)

$u_0 = 0$ $u_0 = \frac{1}{3}(4^0 - 1) = 0$ n=0

$u_n = \frac{1}{3}(4^n - 1)$ P(n)

$u_{n+1} = \frac{1}{3}(4^{n+1} - 1)$ P(n+1)

$u_n = \frac{1}{3}(4^n - 1)$ $u_{n+1} = 4u_n + 1$

$u_{n+1} = \frac{4}{3}.4^n - \frac{4}{3} + 1 = \frac{1}{3}.4^{n+1} - \frac{1}{3} = \frac{1}{3}(4^{n+1} - 1)$ $u_{n+1} = 4\left(\frac{1}{3}(4^n - 1)\right) + 1$

n $u_n = \frac{1}{3}(4^n - 1)$

u_{n+1} u_n n

$ax_0 + by_0 = 1$ $(x_0; y_0) \in$ ² **b a**

$u_{n+1} - 4u_n = 1$ $u_{n+1} = 4u_n + 1$

u_{n+1} u_n $u_{n+1} - 4u_n = 1$ $(1; -4)$

v_0 (v_n) **2**

$v_{n+1} = q.v_n$ n q (v_n)

$v_n = u_n + \frac{1}{3}$ n (v_n)

$v_{n+1} = u_{n+1} + \frac{1}{3} = 4u_n + 1 + \frac{1}{3} = 4u_n + \frac{4}{3} = 4(u_n + \frac{1}{3}) = 4v_n$ $v_n = u_n + \frac{1}{3}$

$v_0 = u_0 + \frac{1}{3} = \frac{1}{3}$ $q = 4$ (v_n)

S_n n

$S_n = v_0 + v_1 + v_2 ..... + v_{3n}$

$S_n = v_0\left[\frac{q^{3n+1} - 1}{q - 1}\right] = \frac{1}{3}\left[\frac{(4)^{3n+1} - 1}{4 - 1}\right] = \frac{1}{9}\left(4^{3n+1} - 1\right)$

$4^{n+1} - 1$ $4^n - 1$ **3**

$3u_{n+1} = 4^{n+1} - 1$ $3u_n = 4^n - 1$ $u_n = \frac{1}{3}(4^n - 1)$

$PGCD(3u_{n+1}; 3u_n) = 3PGCD(u_{n+1}; u_n)$

1 $PGCD(u_{n+1}; u_n) = 1$ $PGCD(4^{n+1} - 1; 4^n - 1) = 3.1 = 3$

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4 n 4^n **7**

$4^3 \equiv 1[7]$ $4^2 \equiv 2[7]$ $4^1 \equiv 4[7]$ $4^0 \equiv 1[7]$

$n =$	$3k$	$3k+1$	$3k+2$
$4^n \equiv$	1	4	2

4^n 7

3

n A_n

$$A_n = 9S_n - 6n - 3^{6n+4}$$

A_n 7 $A_n \equiv 0[7]$

$A_n \equiv 0[7]$ $9S_n - 6n - 3^{6n+4} \equiv 0[7]$

$9S_n = \left(4^{3n+1} - 1\right)$ $9S_n = 3[7]$ $4^{3n+1} \equiv 4[7]$

$3^{6n+4} \equiv \left[(-4)^2\right]^{3n+2}[7]$ $3^{6n+4} \equiv \left[(4)^{3n+2}\right]^2 \equiv 4[7]$

$9S_n - 6n - 3^{6n+4} \equiv 0[7]$ $3 - 6n - 4 \equiv 0[7]$ $6n \equiv 6[7]$

$n \equiv 1[7]$ 6 7 $n = 7p+1$ $p \in$

1 2016 45

1 u_1 u_2 q

(u_n)

$$\begin{cases} \ln(u_1) + \ln(u_2) = 11 \\ u_1 + u_2 = e^4(1+e^3) \end{cases}$$

$$\begin{cases} \ln(u_1) + \ln(u_2) = 11 \\ u_1 + u_2 = e^4(1+e^3) \end{cases} \quad \begin{cases} \ln(u_1).(u_2) = 11 \\ u_1 + u_2 = e^4(1+e^3) \end{cases} \quad \begin{cases} (u_1).(u_2) = e^{11} \\ u_1 + u_2 = e^4(1+e^3) \end{cases}$$

u_2 u_1 $x^2 - Sx + P = 0$ $S = e^4(1+e^3)$ $P = e^{11}$

$x^2 - Sx + P = 0$ $x = u_1 = e^4$ $x' = u_2 = e^7$

$$u_2 = q.u_1 \qquad q = \frac{u_2}{u_1} = e^3$$

2 u_n n

$u_n = u_1.q^{n-1}$ $u_n = e^4(e^3)^{n-1}) = e^{3n+1}$

S_n n

$$S_n = \ln(u_0) + \ln(u_1) + ... + \ln(u_n)$$

$$S_n = \ln(u_0).(u_1).....(u_n) = \ln\left[(u_{0.})^{n+1}.q^{\frac{n(n+1)}{2}}\right] = (n+1)\ln((u_{0.}).q^{\frac{n}{2}})$$

$q = e^3$ $u_0 = e$ $S_n = (n+1)\ln((e).e^{\frac{3n}{2}}) = \frac{(n+1)(3n+2)}{2}$

3 n $PGCD(2S_n; a_n) = PGCD(a_n; 14)$

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$2S_n=(n+1)(3n+2)=3n^2+5n+2 \quad a_n=n+3$

$2S_n-a_n(3n-4)=14 \qquad 2S_n=(n+3)(3n-4)+14=a_n(3n-4)+14$

$PGCD(2S_n;a_n)=PGCD(a_n;14)$ n

$PGCD(2S_n;a_n)$

$PGCD(2S_n;a_n)=PGCD(a_n;14)$

14 7 2 1 14 $PGCD(2S_n;a_n)$

$PGCD(2S_n;a_n)=7$

$n+3=7k \quad PGCD(n+3;14)=7 \quad PGCD(a_n;14)=7 \quad PGCD(2S_n;a_n)=7$

$PGCD(k;2)=1 \quad PGCD(7k;14)=7 \quad PGCD(n+3;14)=7$

$k=2k'+1$ k 2 k $PGCD(k;2)=1$

$k'\in \quad n=14k'+4 \quad k=2k'+1 \quad n=7k-3 \quad n+3=7k$

7 2^n **4**

$2^3\equiv 1[7] \quad 2^2\equiv 4[7] \quad 2^1\equiv 2[7] \quad 2^0\equiv 1[7]$

7 2^n

$n=$	$3k$	$3k+1$	$3k+2$
$2^n\equiv$	1	2	4

3

$\begin{cases} b_n\equiv 0[7] \\ n\equiv 0[5] \end{cases}$ **n** **5**

$b_n=3n.a_n-2S_n+1437^{2016}+1=3n^2+9-(3n^2+5n+2)+1437^{2016}+1$

$2016\equiv 1[3] \quad 1437\equiv 2[7] \quad 1437^{2016}\equiv 2^{2016}[7]\equiv 1[7]$

$n\equiv 6[7] \quad 15n\equiv 6[7] \quad 5n-9\equiv 0[7] \quad b_n\equiv -5n+9[7]$

$2n\equiv 0[35] \qquad \begin{cases} 5n\equiv 0[35] \\ 7n\equiv 0[35] \end{cases} \qquad \begin{cases} n\equiv 0[7] \\ n\equiv 0[5] \end{cases} \qquad \begin{cases} b_n\equiv 0[7] \\ n\equiv 0[5] \end{cases}$

k' $n=35k' \quad n\equiv 0[35]$

7 $1437^{9n+1}-3\times 4^{12n+1}+52$ n **6**

$1437^{9n+1}-3\times 4^{12n+1}+52\equiv 0[7]$ 7 $1437^{9n+1}-3\times 4^{12n+1}+52$

$52\equiv 3[7] \quad 4^{12n+1}\equiv (4^{3n})^4\times 4\equiv 4[7] \quad 1437^{9n+1}\equiv (2^{3n})^3.2\equiv 2[7]$

$1437^{9n+1}-3\times 4^{12n+1}+52\equiv -7[7] \quad 1437^{9n+1}-3\times 4^{12n+1}+52\equiv 2-3.4+3[7]$

$1437^{9n+1}-3\times 4^{12n+1}+52\equiv 0[7]$

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11 7^n 3^n 1

$3^5 \equiv 1[11]$ $3^4 \equiv 4[11]$ $3^3 \equiv 5[11]$ $3^2 \equiv 9[11]$ $3^1 \equiv 3[11]$ $3^0 \equiv 1[11]$

5 11 3^n

n	5k	5k+1	5k+2	5k+3	5k+4
$3^n \equiv$	1	3	9	5	4

$7^6 \equiv 4[11]$ $7^5 \equiv 10[11]$ $7^4 \equiv 3[11]$ $7^3 \equiv 2[11]$ $7^2 \equiv 5[11]$ $7^1 \equiv 7[11]$ $7^0 \equiv 1[11]$

$7^{10} \equiv 1[11]$ $7^9 \equiv 8[11]$ $7^8 \equiv 9[11]$ $7^7 \equiv 6[11]$

10 11 7^n

n	10k'	10k'+1	10k'+2	10k'+3	10k'+4
$7^n \equiv$	1	7	5	2	3
n	10k'+5	10k'+6	10k'+7	10k'+8	10k'+9
$7^n \equiv$	10	4	6	9	8

11 $2\times 2016^{5n+4}+1437^{10n+4}$ n

$2016^{5n+4} \equiv 3^{5n+4}[11] \equiv 4[11]$ $1437^{10n+4} \equiv 3[11]$ $1437^{10n+4} \equiv 7^{10n+4}[11]$

$2\times 2016^{5n+4}+1437^{10n+4} \equiv 0[11]$ $2\times 2016^{5n+4}+1437^{10n+4} \equiv 2.4+3[11]$

$7x-3y=8$ 2

$k \in$ $x=3k+2$ $x \equiv 2[3]$ $7x \equiv 8[3]$ $7x=3y+8$ $7x-3y=8$

$k \in$ $y=7k+2$ $7x=3y+8$ $x=3k+2$

$k \in$ $(x;y)=(3k+2;7k+2)$ $7x-3y=8$

d

$7x-3y=8$ $(x;y)$ y x d

2 $d/3y$ d/y 1 $d/7x$ d/x

$d \in D_8=\{1;2;4;8\}$ $d/8$ $d/7x-3y$ 2 1

d=4

$PGCD(k-2;8)=4$ $PGCD(3k+2;7k+2)=4$

$PGCD(4\alpha;8)=4$ $k-2=4\alpha$

m 2 α $\alpha=2m+1$ $PGCD(\alpha;2)=1$

$(x;y)=(24m+20;56m+44)$ k $k=8m+6$

$2016^{7x}+1437^{3y} \equiv 0[11]$ (E) $(x;y)$

$2016^{7x} \equiv 3^{7x}[11]$ $1437^{3y} \equiv 7^{3y}[11]$

$(x;y) = (3k+2;7k+2)$ $3^{7x} + 7^{3y} \equiv 0[11]$ $2016^{7x} + 1437^{3y} \equiv 0[11]$

10 5 $3^{k} + 7^{k} \equiv 0[11]$ $3^{7x} + 7^{3y} \equiv 0[11]$

k	10p	10p+1	10p+2	10p+3	10p+4
$3^k \equiv$	1	3	9	5	4
$7^k \equiv$	1	7	5	2	3
$3^k + 7^k \equiv$	2	10	3	7	7
k	10p+5	10p+6	10p+7	10p+8	10p+9
$3^k \equiv$	1	3	9	5	4
$7^k \equiv$	10	4	6	9	8
$3^k + 7^k \equiv$	0	7	4	4	1

$n = 10p+5$

p $(x;y) = (30p+17;70p+37)$

1 2015 47

7 2^n **(1**

$2^3 \equiv 1[7]$ $2^2 \equiv 4[7]$ $2^1 \equiv 2[7]$ $2^0 \equiv 1[7]$

7 2^n

3

$n =$	3k	3k+1	3k+2
$2^n \equiv$	1	2	4

7 $1962^{1954} - 1954^{1962} + 2015^{53}$

$1962^{1954} \equiv 2^{1954}[7]$ $1962 \equiv 2[7]$

$1962^{1954} \equiv 2[7]$ $2^{1954} \equiv 2[13]$ $1954 \equiv 1[3]$

$1954^{1962} \equiv 1[7]$ $1954 \equiv 1[7]$

$2015 \equiv -1[7]$ $2015 \equiv 6[7]$

53 $2015^{53} \equiv (-1)[7]$ $2015^{53} \equiv (-1)^{53}[7]$

$1962^{1954} - 1954^{1962} + 2015^{53} \equiv 2 - 1 - 1[7]$

$1962^{1954} - 1954^{1962} + 2015^{53} \equiv 0[7]$

0 7 $1962^{1954} - 1954^{1962} + 2015^{53}$

89 **(2**

9 $\sqrt{89} = 9,4$

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89	2	44	1
89	3	29	2
89	5	17	4
89	7	12	5

89

7832

$7832 = 2^3 \times 11 \times 89$ 7832

$(3+1)(1+1)(1+1) = 16$ 7832

7832

$\times$	11^0	11^1
89^0	1	11
89^1	89	979

$\times$	1	11	89	979
2^0	1	11	89	979
2^1	2	22	178	1958
2^2	4	44	356	3916
2^3	8	88	712	7832

7832

$$D_{7832} = \{1;2;4;8;11;22;44;88;89;178;376;712;979;1958;3916;7832\}$$

977 981

$PGCD(977;981) = 1$ 977 981

1

$981 = 977 \times 1 + 4$

$977 = 4 \times 244 + 1$

977 981

y x 3

$PGCD(x';y') = 1$ $y = 2y'$ $x = 2x'$ $PGCD(x;y) = 2$

$$\begin{cases} (x'-y')(x'+y') = 7832 \\ x'-y' \equiv 4[11] \\ PGCD(x';y') = 1 \end{cases} \quad \begin{cases} x'^2 - y'^2 = 7832 \\ x'-y' \equiv 4[11] \\ PGCD(x';y') = 1 \end{cases} \quad \begin{cases} x^2 - y^2 = 31328 \\ x-y \equiv 8[22] \\ PGCD(x;y) = 2 \end{cases}$$

7832 $x'-y' = 11k+4$

$$\begin{cases} x = 2x' = 1962 \\ y = 2y' = 1954 \end{cases} \quad \begin{cases} x' = 981 \\ y' = 977 \end{cases} \quad \begin{cases} x'-y' = 11k+4 = 4 \\ x'+y' = 1958 \end{cases}$$

$b \times c$ **a** **4**

$(x'_0;y'_0)$ $(x_0;y_0)$ c b a

$ax'_0+cy'_0=1...(2)$ $ax_0+by_0=1...(1)$

$au+(bc)v=1$ $(u;v)$ $b\times c$ a

$(ax_0+by_0)(ax'_0+cy'_0)=1$ 2 1

$a(x_0x'_0+cx_0y'_0+bx'_0y_0)+bc(y_0y'_0)=1$ $a^2x_0x'_0+acx_0y'_0+abx'_0y_0+bcy_0y'_0=1$

$v=y_0y'_0$ $u=x_0x'_0+cx_0y'_0+bx'_0y_0$

$PGCD(a;b^n)=1$ $n\in$ *

$PGCD(a;b)=1$ $n=1$

$PGCD(a;b^{n+1})=1$ $PGCD(a;b^n)=1$

$PGCD(a;b)=1$ $PGCD(a;b^n)=1$

4 $PGCD(a;b^{n+1})=1$

1954^{1962} 1962^{1954}

$PGCD(981^{1954};2^8)=1$ $PGCD(981^{1954};977^{1962})=1$ $PGCD(981^{1954};977)=1$

$PGCD(1962^{1954};1954^{1962}.)=2^{1954}.PGCD(981^{1954};977^{1962}.2^8)=2^{1954}$ 4

1 2014 48

$PGCD(2013;1962)$ **-1**

$PGCD(2013;1962)=3PGCD(671;654)$ $1962=3\times654$ $2013=3\times671$

$PGCD(671;654)=1$

$PGCD(2013;1962)=3$

(E)

$2013x-1962y=54:(E)$

1962y 3 1962 3 2013x 3 2013 3

(E) 54 3 $2013x-1962y$ 3

$x\equiv0[6]$ (E) $(x;y)$

3 (E) $671x-654y=18$ $2013x-1962y=54:(E)$

$671x=6(109y+3)$ $671x=654y+18$ $671x-654y=18$

6 671 $x\equiv0[6]$ $671x\equiv0[6]$

(E) 74 x_0 80 $(x_0;y_0)$

$x_0=78$ 74 x_0 80 $x_0\equiv0[6]$

$671x-654y=18$ $x_0=78$ $y_0=80$ $x_0=78$

$671x-654y=18$ $(78;80)$

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1 2 $\begin{cases} 671x-654y=18...(1) \\ 671(78)-654(80)=18.....(2) \end{cases}$

$671(x-78)=654(y-80).....(*)$ $671(x-78)-654(y-80)=0$

654 671 $(y-80)$ 671 $654(y-80)$ 671 $(*)$

$x=654k+78$ $(*)$ y $y=671k+80$ $(y-80)=671k$

k $(x;y)=(654k+78;654k+80)$ $(x;y)$

d 2

(E) $(x;y)$ y x d

654y d d 671x d x d

$d\in D_{18}=\{1;2;3;4;6;9;18\}$ 18 d $671x-654y$ d

b a

$PGCD(a;b)=18$ $671a-654b=18$ $(a;b)$

$17k+2\equiv 0[18]$ $\begin{cases} 654k+78\equiv 0[18] \\ 671k+80=0[18] \end{cases}$ $\begin{cases} a\equiv 0[18] \\ b=0[18] \end{cases}$

$p\in$ $k=18p+2$ $k\equiv 2[18]$ $-k\equiv -2[18]$

$(a;b)=(11772p+1386;12078p+1422)$ $k=18p+2$

1 2013 49

$PGCD(\alpha;\beta)=PGCD(\beta;10)$ 1

$PGCD(a;b)=PGCD(b;r)$

b a r q $a=b.q+r$

$\beta=n+3$ $\alpha=2n^3-14n+2$

$\alpha=(2n^2-6n+4)\beta+(-10)$ β α

$PGCD(\alpha;\beta)=PGCD(\beta;10)$ $PGCD(\alpha;\beta)=PGCD(\beta;|-10|)$

$PGCD(\alpha;\beta)$

10 d $PGCD(\beta;10)=d$ $PGCD(\alpha;\beta)=d$

$d\in D_{10}=\{1;2;5;10\}$

$PGCD(\alpha;\beta)=5$ n

$n+3=5k$ $\beta=0[5]$ $PGCD(\beta;10)=5$ $PGCD(\alpha;\beta)=5$

$PGCD(k;2)=1$ $5PGCD(k;2)=5$ $PGCD(5k;5.2)=5$

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$p \in$ $n = 10p + 2$ $n = 5(2p+1) - 3$ $k = 2p + 1$ k

k $n = 5k - 3$ $n + 3 = 0[5]$ $\beta = 0[5]$

11 4^n **2**

$4^5 \equiv 1[11]$ $4^4 \equiv 3[11]$ $4^3 \equiv 9[11]$ $4^2 \equiv 5[11]$ $4^1 \equiv 4[11]$ $4^0 \equiv 1[11]$

5 11 4^n

$n =$	$5k$	$5k+1$	$5k+2$	$5k+3$	$5k+4$	
$4^n \equiv$	1	4	5	9	3	$[11]$

$\begin{cases} 4^{5n} + 4^n + n \equiv 0[11] \\ n \equiv 2[10] \end{cases}$ n

$\begin{cases} (4^{5p+1})^5 + (4^{5p+1})^2 + n \equiv 0[11] \\ n \equiv 10p + 2 \end{cases}$ $\begin{cases} 4^{5n} + 4^n + n \equiv 0[11] \\ n \equiv 2[10] \end{cases}$

$(4^{5p+1})^5 \equiv 1[11]$ $(4^{5p+1})^2 \equiv 5[11]$ $4^{5p+1} \equiv 4[11]$

$\begin{cases} n \equiv 5[11] \\ n \equiv 2[10] \end{cases}$ $\begin{cases} (4^{5p+1})^5 + (4^{5p+1})^2 + n \equiv 0[11] \\ n \equiv 10p + 2 \end{cases}$

$n \equiv 82[110]$ $\begin{cases} 10n \equiv -60[110] \\ 11n \equiv 22[110] \end{cases}$ $\begin{cases} n \equiv -6[11] \\ n \equiv 2[10] \end{cases}$

α $n \equiv 110\alpha + 82$

3 2013 50

$2n + 27 \equiv 0[n+1]$ n **1**

$25 \equiv 0[n+1]$ $2n + 2 + 25 \equiv 0[n+1]$ $2n + 27 \equiv 0[n+1]$

$n + 1 \in D_{25} = \{1;5;25\}$ 25 $n+1$ $25 \equiv 0[n+1]$

$n \in \{0;4;24\}$ $n + 1 \in \{1;5;25\}$

$(b-a)(a+b) = 24$ $(a;b)$

24 $(a;b)$

$(3+1)(1+1) = 8$ $24 = 2^3 \times 3$

$D_{24} = \{1;2;3;4;6;8;12;24\}$

$(a;b)$ $(b-a)(a+b) = 24$

$(a;b) = (5;7)$ $\begin{cases} (a+b) = 12 \\ (b-a) = 2 \end{cases}$ $(b-a)(a+b) = 2 \times 12$ 1

$(a;b)=(1;5)$ $\begin{cases}(a+b)=6\\(b-a)=4\end{cases}$ $(b-a)(a+b)=4\times 6$ 2

$\sqrt{24}$

$b^2=a^2+(\sqrt{24})^2$ $b^2-a^2=24$ $(b-a)(a+b)=24$

$7^2=5^2+(\sqrt{24})^2$ $(a;b)=(5;7)$ 1

$BC=7$ $AC=5$ A ABC AB $\sqrt{24}$

$5^2=1^2+(\sqrt{24})^2$ $(a;b)=(1;5)$ 2

$BC=5$ $AC=1$ A ABC AB $\sqrt{24}$

β α 2

$\alpha=1.5^4+0.5^3+1.5^2+4.5^1+1.5^0=671$ $\alpha=\overline{10141}$

$\beta=3.5^3+4.5^2+0.5^1+3.5^0=478$ $\beta=\overline{3403}$

$\begin{cases}b^2-a^2=24\\\alpha a-\beta b=9\end{cases}$ ² $(a;b)$

$\begin{cases}b^2-a^2=24\\671a-478b=9\end{cases}$ $\begin{cases}b^2-a^2=24\\\alpha a-\beta b=9\end{cases}$

$\begin{cases}b^2-a^2=24\\671a-478b=9\end{cases}$ $(a;b)=(5;7)$ 1

$\begin{cases}b^2-a^2=24\\671a-478b=9\end{cases}$ $(a;b)=(1;5)$ 2

$(a;b)=(5;7)$ $\begin{cases}b^2-a^2=24\\\alpha a-\beta b=9\end{cases}$

1434 2013 3

$1434=3.478$ $2013=3.671$

3 1434 2013

478 671

$PGCD(2013;1434)=3.PGCD(671;478)$

23 671 $\sqrt{671}=25,9$ 671

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$PGCD(671;478)=1$

$2013x-1434y=27$ $(x;y)$ ²

$671x-478y=9$ $2013x-1434y=27$

$671x-478y=9$ $(5;7)$ 2

$$\begin{cases}671x-478y=9...(1)\\671(5)-478(7)=9.....(2)\end{cases}$$

$671(x-5)-478(y-7)=0$ 1 2 $\begin{cases}671x-478y=9...(1)\\671(5)-478(7)=9.....(2)\end{cases}$

$671(x-5)=478(y-7).....(*)$

478 671 $(x-5)$ 478 $671(x-5)$ 478 (*)

$y=671k+7$ (*) x $x=478k+5$ $(x-5)=478k$

k $(x;y)=(478k+5;671k+7)$ $(x;y)$

1 2012 51

2011 **1**

44 $\sqrt{62011}=44,84$ 2011

1 1 $(x_0;y_0)$

$$31=579-274\times 2$$
$$31=579-274\times(1432-2\times 579)$$
$$31=579-2\times(1432-2\times 579)$$
$$31=1432(-2)+5\times 579$$
$$31=1432(-2)+5\times(2011-1432)$$
$$31=2011(5)-1432(7)$$

$2011=1432\times 1+579$
$1432=579\times 2+274$
$579=274\times 2+31$

1 $(5;7)$

$$\begin{cases}2011x-1432y=31...(1)\\2011(5)-1432(7)=31.....(2)\end{cases}$$

$2011(x-5)-1432(y-7)=0$ 1 2 $\begin{cases}2011x-1432y=31...(1)\\2011(5)-1432(7)=31.....(2)\end{cases}$

$2011(x-5)=1432(y-7).....(*)$

2011 1432 $(x-5)$ 1432 $2011(x-5)$ 1432 (*)

$y=2011k+7$ (*) x $x=1432k+5$ $(x-5)=1432k$

k $(x;y)=(1432k+5;2011k+7)$ $(x;y)$

7 2^n n 2

$2^3 \equiv 1[7]$ $2^2 \equiv 4[7]$ $2^1 \equiv 2[7]$ $2^0 \equiv 1[7]$

3 7 2^n

$n=$	$3k$	$3k+1$	$3k+2$
$2^n \equiv$	1	2	4

7 $2011^{1432^{2012}}$

$2011^{1432^{2012}} \equiv 2^{1432^{2012}}[7]$ $2011 \equiv 2[7]$

$3k+1$ 1432^{2012} $1432^{2012} \equiv 1[3]$ $1432 \equiv 1[3]$

$2011^{1432^{2012}} \equiv 2[7]$ $2011^{1432^{2012}} \equiv 2^{3k+1}[7]$

2 7 $2011^{1432^{2012}}$

$2010^n + 2011^n + 1432^n \equiv 0[7]$ n

$2010^n \equiv 1[7]$ $2010 \equiv 1[7]$ $2011^n \equiv 2^n[7]$ $2011 \equiv 2[7]$

$1432^n \equiv (2^n)^2[7]$ $1432 \equiv 4[7]$

$1+2^n+2^{2n} \equiv 0[7]$ $2010^n + 2011^n + 1432^n \equiv 0[7]$

n

$n=$	$3k$	$3k+1$	$3k+2$
$2^n \equiv$	1	2	4
$2^{2n} \equiv$	1	4	2
$2^{2n}+2^n+1 \equiv$	3	0	0

$n = 3k+2$ $n = 3k+1$ n

N γ β α

9 $\overline{2\gamma\alpha\beta}$ N

$2011\beta - 1432\gamma = 31$ 1 $(\beta;\gamma)$

$0 \le \gamma \le 8$ $0 \le \beta \le 8$ $\gamma = y = 2011k+7$ $\beta = x = 1432k+5$

$k = 0$ $0 \le 2011k+7 \le 8$ $0 \le 1432k+5 \le 8$

$\gamma = 7$ $\beta = 5$

γ β α

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$\alpha = 2\beta - \gamma = 3$ γ β α 3β

$$N = \overline{2\gamma\alpha\beta} = \overline{2735} = 2.9^3 + 7.9^2 + 3.9 + 5.9^0 = 2057$$

1 2011 52

(E) 1

y x $13x - 7y = -1 \ldots (E)$

(E)

$13 \equiv -1[7]$ $-x \equiv -1[7]$ $13x \equiv -1[7]$ $13x = 7y - 1$ (E)

k $x = 7k + 1$ $x \equiv 1[7]$

$y = 13k + 2$ $13x = 7y - 1$ x

k $(x; y) = (7k + 1; 13k + 2)$

$\begin{cases} a \equiv -1[7] \\ a \equiv 0[13] \end{cases}$ a 2

$a \equiv 13[91]$ $6a \equiv -13[91]$ $\begin{cases} 13a \equiv -13[91] \\ 7a \equiv 0[91] \end{cases}$ $\begin{cases} a \equiv -1[7] \\ a \equiv 0[13] \end{cases}$

p $a \equiv 91p + 13$

(E)

13 7 9^n n 3

7 9^n n

$9^3 \equiv 1[7]$ $9^2 \equiv 4[7]$ $9^1 \equiv 2[7]$ $9^0 \equiv 1[7]$

3 7 9^n

n =	3k	3k + 1	3k + 2	
$9^n \equiv$	1	2	4	[7]

13 9^n n

$9^3 \equiv 1[13]$ $9^2 \equiv 3[13]$ $9^1 \equiv 9[13]$ $9^0 \equiv 1[13]$

3 13 9^n

n =	3k'	3k'+1	3k'+2	
$9^n \equiv$	1	9	3	[13]

91 b β α 4

$$b = \overline{\alpha 00\beta 086} = \alpha.9^6 + \beta.9^3 + 8.9 + 6$$

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13 7 $91=7.13$ $\begin{cases} b\equiv 0[7] \\ b\equiv 0[13] \end{cases}$ 91 b

$78\equiv 1[7]$ $9^3\equiv 0[13]$ $9^6\equiv 0[13]$ $9^3\equiv 0[7]$ $9^6\equiv 0[7]$ $\begin{cases} \alpha+\beta+1\equiv 0[7] \\ \alpha+\beta\equiv 0[13] \end{cases}$

$\alpha+\beta\equiv 91p+13$ 2

$p=0$ $0\le 91p+13\le 16$ 0 $\beta\le 8$ 0 $\alpha\le 8$

91 b

$(\alpha;\beta)=(7;6)$ $(\alpha;\beta)=(6;7)$ $(\alpha;\beta)=(8;5)$ $(\alpha;\beta)=(5;8)$

1 2010 53

13 $3^{3n}-1$ $n\in$ 1

$3^{3n}\equiv 1[13]$ $n\in$ $3^3\equiv 1[13]$

13 $3^{3n}-1$ $n\in$

13 $3^{3n+2}-9$ $3^{3n+1}-3$ n 2

$3^{3n+1}-3\equiv 0[13]$ $3^{3n+1}\equiv 3[13]$ $3^{3n}\equiv 1[13]$ $n\in$

$3^{3n+2}-9\equiv 0[13]$ $3^{3n+2}\equiv 9[13]$ $3^{3n}\equiv 1[13]$ $n\in$

13 3^n n 3

$3^3\equiv 1[13]$ $3^2\equiv 9[13]$ $3^1\equiv 3[13]$ $3^0\equiv 1[13]$

3 13 3^n

$n=$	$3k'$	$3k'+1$	$3k'+2$	
$3^n\equiv$	1	3	9	[13]

13 2005^{2010}

$2005^{2010}\equiv 1[13]$ $2005^{2010}\equiv 3^{3.670}[13]$ $2010\equiv 0[3]$ $2005\equiv 3[13]$

1 13 2005^{2010}

$p=3n$ 13 A_p 4

$A_{3n}=3^{3n}+3^{2(3n)}+3^{3(3n)}$ $A_p=3^p+3^{2p}+3^{3p}$

$A_{3n}\equiv 3[13]$ $3^{3(3n)}\equiv 1[13]$ $3^{2(3n)}\equiv 1[13]$ $3^{3n}\equiv 1[13]$

13 A_p $p=3n+1$

$A_{3n+1}=3^{3n+1}+3^{2(3n+1)}+3^{3(3n+1)}$

$A_{3n+1}\equiv 0[13]$ $3^{3(3n+1)}\equiv 1[13]$ $3^{2(3n+1)}\equiv 9[13]$ $3^{3n+1}\equiv 3[13]$

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$p=3n+2$ 13 A_p

$$A_{3n+2}=3^{3n+2}+3^{2(3n+2)}+3^{3(3n+2)}$$

$A_{3n+2}\equiv 0[13]$ $3^{3(3n+2)}\equiv 1[13]$ $3^{2(3n+2)}\equiv 3[13]$ $3^{3n+2}\equiv 9[13]$

0 $p=3n+2$ 13 A_p

A_p b a

$a=A_3$ $a=\overline{1001001000}=1.3^9+1.3^6+1.3^3$

$b=A_4$ $b=\overline{1000100010000}=1.3^{12}+1.3^8+1.3^4$

13 b a

3 13 a $a\equiv 3[13]$ $A_3\equiv 3[13]$ $a=A_3$

0 13 b $b\equiv 0[13]$ $A_4\equiv 0[13]$ $b=A_4$

2 2010 54

7 y 1 $(x;y)$ 1

$65y=7(287-x)$ $65y=2009-7x$ $7x+65y=2009....(1)$

7 65 $y\equiv 0[7]$ $65y\equiv 0[7]$

7 y $y\equiv 0[7]$

1

k $y=7k$ 7 y

$x=65k+287$ $65y=7(287-x)$ y

k $(x;y)=(-65k+287;7k)$ $(x;y)$

9 2^n n 2

$2^6\equiv 1[9]$ $2^5\equiv 5[9]$ $2^4\equiv 7[9]$ $2^3\equiv 8[9]$ $2^2\equiv 4[9]$ $2^1\equiv 2[9]$ $2^0\equiv 1[9]$

6 9 2^n

$n=$	$6k$	$6k+1$	$6k+2$	$6k+3$	$6k+4$	$6k+5$
$2^n\equiv$	1	2	4	8	7	5

9 $2^{6n}+3n+2$ n 3

$2^{6n}+3n+2\equiv 0[9]$ 9 $2^{6n}+3n+2$

$3n\equiv -3[9]$ $1+3n+2\equiv 0[9]$ $2^{6n}+3n+2\equiv 0[9]$

p $n\equiv 3p+2$ $n\equiv 2[3]$

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9 u_n 4

$u_n = 2^{6n} - 1$

$u_n \equiv 0[9]$ $2^{6n} - 1 \equiv 0[9]$ $2^{6n} \equiv 1[9]$ 2

9 u_n

$(7u_1)x + (u_2)y = 126567....(2)$

$7x + 65y = 2009....(2)$ $(7u_1)x + (u_2)y = 126567....(2)$

$126567 \equiv 2009[9]$ $u_2 \equiv 65[9]$ $u_1 \equiv 1[9]$

1 2

$y_0 \geq 25$ y_0 x_0 2 $(x_0; y_0)$

$\frac{25}{7} \leq k \leq \frac{287}{65}$ $-65k + 287 \geq 0$ $7k \geq 25$ $x_0 \geq 0$ $y_0 \geq 25$

$(x_0; y_0) = (27; 28)$ $k = 4$

1 2009 55

y x $(5x^2 + 6)(x + 1)$ 1

$(5x^2 + 6)(x + 1) = 5x^3 + 5x^2 + 6x + 6 = A....(1)$

$A = (5x^2 + 6)(2 + 2y).....(2)$

$1 + 2y = x$ $2 + 2y = x + 1$ 2 1

12 x y x

11 7 x 12 7 x

$x = 7$ $y = 3$ $1 + 2y = x$

$x = 11$ $y = 5$

A

$A = (5.7^2 + 6)(2 + 2.3) = 2008$ $y = 3$ $x = 7$ 1

$A = (5.11^2 + 6)(2 + 2.5) = 7332$ $y = 5$ $x = 11$ 1

584 2

$584 = 1^2.2^{2.}.2.73$

2 1 584

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$\begin{cases} a+b=32 \\ a^2+b^2=584 \end{cases}$ a b b a

$PGCD(a';b')=1$ $b=d.b'$ $a=d.a'$ $PGCD(a;b)=d$

$$\begin{cases} a'+b'=\dfrac{32}{d} \\ a'^2+b'^2=\dfrac{584}{d^2} \end{cases} \qquad \begin{cases} a+b=32 \\ a^2+b^2=584 \end{cases}$$

$d=2$ $d=1$

$\begin{cases} b'=32-a' \\ a'^2-32a'+220=0 \end{cases}$ $\begin{cases} a'+b'=32 \\ a'^2+b'^2=584 \end{cases}$ $d=1$ 1

22 10 $(22;10)$ $(10;22)$

$\begin{cases} b'=16-a' \\ a'^2-16a'+55=0 \end{cases}$ $\begin{cases} a'+b'=16 \\ a'^2+b'^2=146 \end{cases}$ $d=2$ 1

11 5 $(5;11)$ $(11;5)$

$(a;b)=(22;10)$ b a

2 2008 56

^{2} (E) 1

$PGCD(3;21)=3$ $3x-21y=78....(E)$

78 3 $3x-21y=78....(E)$

$x\equiv 5[7]$ (E) ^{2} $(x;y)$

$x=26+7y$ $x-7y=26....(E')$ (E)

$26\equiv 5[7]$ $x\equiv 5[7]$ $x\equiv 26[7]$ $x=26+7y$

(E)

k $x\equiv 7k+5$ $x\equiv 5[7]$

$y=k-3$ $x-7y=26....(E')$ $x\equiv 7k+5$

k $(x;y)=(7k+5;k-3)$ $(x;y)$

7 5^n n 2

$5^6\equiv 1[7]$ $5^5\equiv 3[7]$ $5^4\equiv 2[7]$ $5^3\equiv 6[7]$ $5^2\equiv 4[7]$ $5^1\equiv 5[7]$ $5^0\equiv 1[7]$

6 7 5^n

$n=$	$6p$	$6p+1$	$6p+2$	$6p+3$	$6p+4$	$6p+5$
$5^n\equiv$	1	5	4	6	32	3

$5^x + 5^y \equiv 3[7]$ (E) ² $(x;y)$

3 k $(x;y) = (7k+5;k-3)$ ² $(x;y)$ (E)

k' $k' = k-3$

$(x;y) = (7k'+21;k')$ ² $(x;y)$ (E)

$5^{7k'+26} + 5^{k'} \equiv 3[7]$ $5^x + 5^y \equiv 3[7]$

$5^{6k'}.5^{k'}.5^{6.4+2} + 5^{k'} \equiv 3[7]$ $5^{7k'+26} + 5^{k'} \equiv 3[7]$

$5^{6.4+2} \equiv 4[7]$ $5^{6k'} \equiv 1[7]$ $5^{k'+1} \equiv 3[7]$

$k' = 6p+4$ $k'+1 = 6p+5$

p $(x;y) = (42p+54;6p+4)$ k'

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